MODEL OF FIRM IN TRANSIENT ECONOMY

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Summary

The model of behavior of industrial firm with a possibility of merchandising and purchases of production on two channels «traditional» and «commercial». The former is stable, but low profitable due to non-payments. The later is profitable, but risky. The model describes different modes of firm operation depending on economic parameters. In such model the firms have incentives to integration in financial and industrial groups.

1. Introduction

The relatively stable state of the Russian economy in 1995-1998 looks something strange from the point of view of the mainstream economic theory. The state is characterized by permanent, but not rapid decrease in production, coexistence of several types of money, unusual functions of banks in conditions of lack of the investments and low profitability of production [1].

In the present paper we offer the description of activity of a firm which attempts to promote its production on the market, but often fail and even being successful bear substantial transaction costs. This results in serious problems with working capital of the firm. When such firms forms majority, they could try to solve their problems by interfirm market with special prices. The model logically includes the non-payments and promissory notes circulation as the special types of money. The model also displays an essential role of fixed costs and also explains the tendency to spontaneous integration of firms under aegis of banks in a certain similarity of financial-industrial groups (FIG). The model such a FIG is the kernel of model of regional economy explained in [2].

The problem of the non-payments is in a center of attention of Russian economists and the Western ones, interested in transition economy appearance. The scales and reasons of origin of the non-payments are considered in [3,6]. In [4J the case study of debts will be carried out from the point of view of a financial state of firm. In [5] the possibility of struggle with the non-payments is considered through implantation of a system of circulation of promissory notes. It is necessary also to mark [7], where the role of barter in modern Russian economy is considered. From the point of view adopted here barter and the non-payments fulfill similar functions, creating additional possibilities for decreasing transaction costs (including expenditures on fulfilling liquidity constrains).

Much less publications are devoted to the non-payments within the framework of formal models. We can point out only [8-12J. The authors everyone in own way attempt to answer a problem, why a firm prolongs to deliver production, knowing, that it will not be completely paid. Papers [8, 9] consider the situation of the first years of an economic

reform in Russia, when the firms relied on the state preferential credits and stored the non-payments as a justification of their getting. In [10, 11] the model of a general equilibrium in economy with the non-payments was proposed. In [12] the non-payments are parsed from the microeconomic point of view - the model of interaction of the buyer - seller with the asymmetrical information is considered. In [8-12] the equilibrium with the non-payments occurs to be effective.

In the present work we do not consider general equilibrium model, parsing behavior of firm at given to system of non-payments. However at the end of work we show some reasons, how on the basis of the offered model it is possible to construct model of a macroeconomic equilibrium with the non-payments. Besides we pay special attention to economy with fixed costs and we display that in this case system with the non-payments is somewhat more preferable.

2. Model of Industrial Firm with Two Channels of Delivering

Let's consider industrial firm producing homogeneous good. As the factors of production we shall take into account only material working capitals (raw material and components). Such description meets to a present state of matters in the Russian economy, when the manpower in industry is surplus and the fix capital practically do not vary. We consider stationary model of activity of firm, thus beforehand excepting investments (that also meets to an observable reality).

The technological possibilities of such firm are described by the cost function $v = \psi(x)$, where \tilde{o} - gross output, and v material input per unit of time. As we will research only financial state of firm (incomes, cost, profits, and credits) under given constant prices, both input and output may be assumed to be scalars. Suppose that the cost function $\psi(x)$ is defined for all $\tilde{o} > 0$, is monotonic, strictly concave, $\psi'(x) = 0$, $\psi'(\infty) = \infty$, $\psi(0) > 0$. (Usually suppose $\psi(0) = 0$, but we shall pay special attention to a role of stationary values of costs $\psi(0)$.)

Our main supposition is, that the firm has two channels to purchase row material and to deliver output. The first channel we will refer to as «traditional», the second as «commercial».

- 1. The traditional channel corresponds to delivering by the direct inter-firm links, according with firm's specialization in the Soviet economy. Data of inquiries [13,14] display, that direct links cover about 75-85 % of delivering. To the traditional channel we also refer delivering production to the government. We guess that the traditional channel is characterized by the following properties.
 - The delivering by direct links links are stable enough: the firm can choice levels of delivering and purchases on the traditional channel, but being once chosen, these levels should be withstood long enough (within the framework of model always).

- Delivering and the purchases on direct links admit and even guess the non-payments. As well as in [9], we consider the non-payments as the special type of money with a particular system of the prices. For production, sold by firm, and expended raw material we denote these prices by δ_1 and δ_2 for production of the firm and for row material respectively. These prices have the the following meaning: if a market price of production is q_1 , then unit of good delivered through traditional channel gives the firm $q_1 \delta_1$ units of money and increments of her accounts receivable by δ_1 . Similarly if q_2 is market price of raw material, the firm pays only $q_2 \delta_2$ in money and increments her accounts payable by δ_2 .
- 2. Commercial channel. In present conditions the majority of industrial firms tend to adapt to the market: the firms search for new types of production (sometimes more primitive, than the main one), which can be sold for "alive" money. In general these attempts could hardly considered to be successful. Therefore we describe outcome of such attempts as stochastic process. In model this risky commercial channel is characterized by the following properties.
 - The firm not always has a possibility of sales through commercial channel. This possibility occurs and fades by a random, i.e. in each moment of time the firm can be in one of two states: «Ò» when it can use only traditional channel and «C» when it delivers production on both channels. The state «T» turns into the state «C» with frequency λ and «C» turns into «T» and with frequency μ . Thus, the average time, which firm spends in the state «C», equals to $\tau = \lambda / (\lambda + \mu)$.
 - The purchases of raw materials through commercial channel are paid by market price q_2 .
 - It is known, that the industrial firms in Russia have acquired with a great many of intermediaries and that the trade margin in the Russian economy is held on enough high level. So we assume, that, delivering through the commercial channel Z units of good per unit of time, the firm gains only $q_1 Z \varphi(Z)$, where $\varphi(Z)$ the strictly convex selling cost function $(\varphi(0) = 0, \varphi'(0) = 0, \varphi'(\infty) = \infty, \varphi''(Z) > 0)$. The convexity of $\varphi(Z)$ reflects the fact, that at the unexpectedly arisen need to sell a major batch of production pushes specific selling costs above, the level corresponding to stable sales of small batches.

Thus, the traditional channel may supply firm with cheap raw materials and admit low-profitable, but stable sales of her product. The commercial channel is in average more profitable, but unstable. Underline, that we do not suppose any quotas on sales through commercial channel. Even at small τ the firm can store her product in the state «T» and implement large average sales τ Z through commercial channel in the state «C». Since the firm may ignore the commercial channel when it is available, the state «C», when the commercial channel is open, at any rate is not less preferable, than state «O», when the commercial channel is unavailable.

Let's pass to formal description of activity of the firm. Denote by Q_1 and Q_2 a store of finished product and raw materials respectively. Then

$$dQ_1/dt = x_T - Y$$
 in «T»

$$dQ_2/dt = x_C - Y - Z$$
 in «C»

where Y denote delivering through the traditional channel, not depending on state, on the; Z — delivering through the commercial channel, x_T , x_C — levels of the firm's production in the states «T» and «C» respectively.

$$dQ_2/dt = -v_T + V$$
 in «T»

$$dQ_2/dt = -v_C + V + W$$
 in «C»

where V denote purchases of raw materials trough the traditional channel, not depending on a state, W – purchases on the commercial channel in the state «C», V_T , V_C – levels of material input in the states «T» and «C» respectively.

Let's pass now to financial balances of the firm. The money deposit \hat{I} , (current account) varies in time as follows:

$$dM/dt = -B_T + (q_1 - p_1) Y - (q_2 - p_2) V + K_T - H_T \qquad in \ (T)$$

$$dM/dt = -B_C + (q_1 - p_1) Y - (q_2 - p_2) V + K_C - H_C - q_2 W + q_1 Z - \varphi(Z) \qquad in \ (C)$$

where B_s denote gross income of the firm in the state s = T, C. The gross income covers profit, wage bill, fixed expenditures, taxation and so on. K_s , I_s denotes bank credit and debt servicing in the state s = T, C; q_1 , q_2 – market prices and p_1 , p_2 . – «rates of non-payment» for sold good and purchased row materials respectively.

Accordingly debtors receivable S_A and accounts payable S_P of the firm grow as

$$d S_A/dt = p_1 Y;$$
 $d S_P/dt = q_2 V$

These relations mean, that inter-firm debts are never paid off. Comparison to empirical data for 1995-1997 allows to consider such supposition as quite admissible.

Debt servicing I_s consist of interest payments and return of previous credits, so the debts to the bank are paid off at least partially. Dynamic of firm's debt to bank depends on the received mode of extra charge of interest rate and is out of interest for us. We simply assume as in [1] that the bank crediting the firm gains the constant average interest r

(2)
$$\tau H = (1+r)(1+\tau)K$$
 $H = \mathbf{E}\{H_C - K_C \mid \text{«C»}\}; K = \mathbf{E}\{K_T - H_T \mid \text{«T»}\}$

While labor is not limiting factor of production and the investments are practically miss, the subdivision of gross income on profit, wage bill and amortization does not influence the choice of optimum firm policy. So we guess, that the firm is interested in the gross income. In our model the gross income is stochastic. As in [1] we guess, that the bank crediting firm, taking all risk of losses in a "poor" state «T», as long as it is prolonging. Then it is logical to assume the firm to be risk neutral, that is to assume that she is interested in the average gross income.

Further we will consider only the stationary case when the values of control variables x_T , x_C , v_T , v_C , Y, V, Z, W, K_T , H_T , K_C , H_C remain constant and phase variables M, Q_1 , Q_2 fluctuate around their constant average values. Under these assumptions the optimal behavior of the firm should maximize the value of average income

(3)
$$J = \tau B_C + (1-\tau) B_T \implies max$$

over control variables x_T , x_C , v_T , v_C , Y, V, Z, W, K_T , H_T , K_C , H_C subject to constraints which may be divided into three groups.

1. <u>Technological constraints.</u>

(4)
$$v_T \ge \psi(x_T)$$
; $v_C \ge \psi(x_C)$; $x_T \ge 0$; $x_C \ge 0$

2. <u>Constraint of a nonnegativity of the material stocks</u>. In the stationary process the necessary condition of nonnegativity of stocks is a condition of a nonnegativity of mean values of their derivative over time

(5)
$$\tau x_C + (1-\tau) x_T - \tau Z - Y \ge 0; \qquad -\tau v_C - (1-\tau) x_T + \tau W + V \ge 0$$

Besides that, we demand, that in a state «O» the firm fulfilled her the obligations on direct links with probability 1. Since the state «O» may be prolonged unrestrictedly, necessary and sufficient condition of fulfilling obligations are the following inequalities

$$(6) x_T \ge Y; V \ge v_T$$

We do not put such a restriction for the state «C» considering, that in a commercial condition the firm can operatively cover possible shortage of input caused by unexpectedly long functioning of the commercial channel. Of course, it causes additional expenditures. Their mean value should be accounted in the sale cost function φ .

<u>Institutional restrictions</u> We exclude a possibility to sell goods for money beside trade intermediaries. This limitation are expressed by the following inequalities.

$$(7) W \ge 0; Z \ge 0$$

Really, negative W would mean that the firm sells raw material in the market, gaining complete price q_2 but does not pay sale cost $\varphi(-W)$. The second restriction in (7) was put

to remove the question of prolongation of the function $\varphi(Z)$ on negative values. At any reasonable setting of problem of the firm behavior the solution Z should be nonnegative.

<u>Solvency constraint</u>. Under assumption made above up to now any firm would prefer to purchase raw materials *V* through the traditional channel for non-payments and deliver her product on market for full price. However such behavior would break equilibrium. We guess, that inter-firm relation imply some balance between payable and receivable accounts. A firm justifies her own non-payments to suppliers by non-payments of her consumers. In other words accounts payable (liabilities) are ensured by accounts receivable (assets). In the model this balance can be entered as follows:

$$(8) p_2 V \leq p_1 Y + \delta$$

where δ is prescribed from the outside acceptable value of «additional emission of the non-payments» for the given firm. The most natural value for δ is 0.

<u>Liquidity constraint</u>. As well as in case of material stocks, the requirements that the current account will be nonnegative with probability 1 for stationary process reduces to the conditions of a nonnegativity of cash flows $d\vec{l}/dt$ in both states. Since the firm maximizes a functional (3), it is obvious, that B_T and B_C are determined from conditions $\vec{l} = 0$ in the respective states (see (1)). Thus it is necessary to demand, that the gross income in both states was nonnegative B_T , $B_C \ge 0$. Since the state «C» deliver additional possibilities in comparison with the state «T» B_C any rate is not less, than B_T , therefore it is enough to impose a condition of a nonnegativity only on B_T . Using (1), (2) inequality $B_T \ge 0$ may be rewritten as

(9)
$$(q_1 - p_1) Y - (q_2 - p_2) V + K \ge 0$$

We also should prohibit the firm to gain money by crediting bank. In view of (2) this means that

(10)
$$K \ge 0$$

So we will describe the firm behavior as the solution of the following optimization problem

(11)
$$J = (q_1 - p_1) Y - (q_2 - p_2) V + \tau (q_1 Z - \varphi(Z) - \text{subject to (4) - (10)}$$
$$q_2 W) - r (1 - \tau) K \Rightarrow max$$

<u>Proposition 1</u>. Let the set of admissible values of control variables is not empty¹, i.e. there exist x_C , x_T , v_C , v_T , Y, V, Z, W and K, satisfying (4) - (10). Then the problem of maximization of the functional (11) over x_C , x_T , v_C , v_T , Y, V, Z, W, K subject to constrains (4) - (10) has unique finite solution. On this solution conditions (4), (5) are fulfilled as equalities, and

¹ This requires harmony between fixed costs $\psi(0)$ and quota of the non-payments Δ .

(12)
$$K = max\{0, -(q_1 - p_1) Y + (q_2 - p_2) V\}$$

We have assumed that the firm maximizes gross income at the given interest rate r. Let's compare this problem with the problem in which the firm has no possibility to take credit, i.e. when K = 0 and the liquidity constraint (9) looks like

(9)
$$(q_1 - p_1) Y - (q_2 - p_2) V \ge 0$$

The problem (11) subject to (4) - (8), (9) also has a unique solution. Let λ be the Lagrange multiplier removing liquidity constraint in the problem without credit. It is easy to show (see[1]) that if $r > r^* = \lambda/(1-\tau)$, then in the initial problem (11) subject to (4) - (10) the firm will not take the credit (K = 0). For this reason the bank certainly will not set interest rate higher than r^* . Therefore further we will solve the problem of behavior of firm only at $r \in [0, r]$.

Further it will be convenient to include solvency constraint (8) in the functional J with Lagrange multiplier θ and consider Lagrangian

(13)
$$L(\theta) = (q_1 - p_1) Y - (q_2 - p_2) V + \tau (q_1 Z - \varphi(Z) - q_2 W) - r (1 - \tau) K + \theta (p_1 Y + \delta - p_2 V)$$

According to saddle-point theorem there exist $\theta_* \ge 0$ such that

(14)
$$\max_{\theta} J = \max_{\theta} L(\theta_*) \le \max_{\theta} L(\theta)$$
 for all $\theta \ge 0$

where max_0 denotes maximization over x_C , x_T , v_C , v_T , Y, V, Z, W and K subject to (4) - (10) and max_1 denotes maximization over the same variables subject to (4) - (7), (9), (10), that is without solvency constraint. When $\theta = \theta_*$ solutions of problems $max_0 J$ and $max_1 L(\theta_*)$ coincide. The value θ_* may by interpreted as the inner rate of non-payments of the firm.

3. Non-payments and Integration of Firms

The offered model of behavior of firm allows to reveal reasons of joining of firms in formal or informal financial-industrial groups (FIG). By definition FIG carries "the joint liability over obligations". Let's consider a group N of different firms, the behavior of each is described by the model stated above. Within the section we will mark values, corresponding to different firms by superscript $j \in N$. The joint liability over obligations can be understood as change individual solvency constraints $p_2 V^j \le p_1 Y^j + \delta^j$ (see (9)) by the total one

(8)
$$\sum_{i \in N} p_2 V^j \leq \sum_{i \in N} p_i Y^j + \sum_{i \in N} \delta^j$$

It is possible to consider a problem of maximization of the aggregate gross income of FIG $J^N = \sum_{j \in N} J^j$ subject to individual constraints (4) - (7), (9), (10) and total solvency

constraint (8). Similarly to the problems for each firm, this problem for all also has a unique finite solution. Let Θ_* be Lagrange multiplier to constraint (8) in aggregate problem. From (14) we obtain that

(15)
$$\max_{0} J^{N} = \sum_{j \in N} \max_{1} L^{j}(\Theta_{*}) \ge \sum_{j \in N} \max_{1} L^{j}(\theta_{*}^{j}) = \sum_{j \in N} \max_{0}$$

this means that gross income of FIG exceeds the some of their individual incomes and hence aggregation into FIG is profitable for individual firms.

The obtained result (15) can be interpreted as follows. We consider firm operating in economy with two systems of the prices and accordingly with two types of money: usual money and non-payments. At lack of the open market, on which one money can be exchanged for another, the Greshem's law does not hold, and each firm appreciates the non-payments differently: magnitude of inner rate of non-payments θ^{*j} depends on τ^{j} , $\psi^{j}(\cdot)$, $\varphi^{j}(\cdot)$ etc. and are not obliged to coincide for different firms. The toting of solvency constraints is equivalent to organization of the local competitive market of non-payments, on which the firms with smaller θ^{*j} sell and firms with major θ^{*j} buy non-payments at equilibrium price Θ^{*} . In the absence of such a market non-payments (the mutual credit of firms) circulate only along technological chains. The formation of the market of non-payments allows to spread the mutual credit to firms irrelevant with each other technologically.

Instead of organization of the local market it is possible to achieve the same result by reallocation of quotas on "emission" δ^j inside FIG. It will be enough to change δ in (8) for the firm $j \in N$ by

$$\Delta^{j} = p_1 Y_*^{j} - p_2 V_*^{j}$$

where Y_*^j , V_*^j are the solutions of the problem $max_0 J^N$. Easy to prove that it is real reallocation of quotas,

$$\sum_{j \in N} \Delta^{j} = \sum_{j \in N} \delta^{j}$$

In any case - whether by organization of the local market, whether by reallocation of quotas - the manager of the market (central company or bank) is necessary for the FIG. The most natural mode of reallocation of debts is well-known the financial procedure of the rediscount of promissory notes by bank, which consists in the following. The firm wishing to sell for money debt receivable, beforehand interchanges it for the banking promissory note, which is bought by other firm (in practice it more often buyer of production of debtors). The sense of this procedure that the banking promissory note is

ensured by assets of banks, instead of the chain of mutual non-payments. The functions of bank are to test and to satisfy by promissory note the actual solvency of debtor.

Actually for services in such organization of the mutually advantageous agreements the intermediaries (in particular, the banks) raise particular bill discount. In the model we neglect it and guess, that bank operate disinterestedly.

We speak above only of solvency constraints (8). Reallocations of financial flows within FIG can also soften liquidity constraints (9). As it was shown in [1] It can be made by assigning individual interest rates to the firms.

4. Optimum Behavior of the Firm

The above results shows that the behavior the firm included in FIG may be described as the solution of the following optimization problem (we again omit an index of the firm *j*):

(16)
$$J = s_1 Y - s_2 V + \tau (q_1 Z - \varphi(Z) - q_2 W) \Rightarrow max \quad subject to (4) - (7), (9)$$

where

$$s_1 = (q_1 - p_1) (1 + r (1 - \tau)) + \Theta_*$$
 $s_2 = (q_2 - p_2) (1 + r (1 - \tau)) + \Theta_* p_2$
 p_1 ;

are the effective prices of delivering and purchases through the traditional channel, Θ * is the equilibrium rate of non-payments inside the given FIG.

<u>Proposition 2</u> Let cost functions $\psi(\cdot)$ and $\varphi(\cdot)$ obey to the made above suppositions. Then the solution of the problem (16) is characterized as follows.

If $s_1/q_1 < 1$ and $s_2/q_2 \le 1$, then W = 0, Z, $x_T > 0$, $V = v_C = v_T$. $= \psi(x_T)$, $x_C = x_T$, $O(x_T) = x_T = v_T$. $v_T = v_T$ and the values of $v_T = v_T$ are determined by the equations

$$\varphi'(Z) = q_1 - s_1, \ \psi'(\tilde{o}_T) = s_1 \ / \ s_2.$$

1. If $s_1/q_1 \ge 1$ and $s_1/q_1 \ge s_2/q_2$, then Z = W = 0, $O = x_C = x_T$, $V = v_C = v_T = \psi(x_T)$ and the value of x_T is determined by the equation

$$\psi'(\tilde{o}_T) = s_1 / s_2.$$

2. If $s_2/q_2 \ge 1$ and $s_2/q_2 \ge s_1/q_1$, then $W = v_C - v_T > 0$, Z > 0, $V = v_T$, $v_C = \psi(x_C) > v_T = \psi(x_T)$, $Y = x_T - \tau$ ($Z + x_T - x_C$) and the values of Z, x_C , x_T are determined by the equations

$$\varphi'(Z) = q_1 - t_1, \quad \psi'(\tilde{o}_T) = (s_1 - \tau t_1) / (s_2 - \tau q_2), \quad \psi'(\tilde{o}_C) = t_1 / q_2.$$

Here parameter t_I is defined by the following algorithm; if at $t_I = s_I$ the relations above give the values such that $Z + x_T - x_C > 0$ then put $t_I = s_I$, else find $t_I < s_I$ so that $Z + x_T - x_C = 0$.

The proposition 2 shows that the model allows qualitatively different policies of the firm. The case 1 describes the firm, inside which both channels are mixed with each other. Produced goods are delivered trough both channels, but all row materials are purchased thorough traditional one only.

The case 2 corresponds to the firm using the traditional channel exclusively. The case 3 describes the firm, inside which coexist, not being mixed, market and traditional structures. Part of goods directed in the traditional channel is being made with row materials purchased through the traditional channel. Part of goods directed in the commercial channel is being made with row materials purchased for money at the market.

Inquiries of the principals of firms display, that at modern Russian economy there are presented all three modes of policy, but the most typical is the case 1. The case 1 also is symmetric concerning relations of the prices s_i/q_i . i=1,2. It is possible to imagine the model of economy, in which all firms operate as in the case 1, delivering one part of production each other trough the traditional channel, and other part - trough the commercial channel on the consumer market.

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