Self-Organization of Trade Networks in an Economy with Imperfect Infrastructure

Sergei Guriev, Igor Pospelov and Margarita Shakhova
Computing Center of Russian Academy of Science

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Abstract

A multi-agent model is proposed for analysis of self-organization of trade networks. The model takes into account time spent on transactions ("trade distance"). It is shown that the same set of traders may generate trade networks of different structures depending on average trade distance. When the latter is small, the market is near-competitive. When trade distance is large, the set of traders exhibits monopolistic behavior. Under medium trade distance a phase transition and complex dynamics are observed, including significant price oscillations, regular bursts of shortages and long chains of traders. Emergence of trader's market strategies such as stabilizing wholesale traders and destabilizing speculators is discovered. The model is studied both analytically and via computer simulations.

[1]

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1 Introduction

The problem of self-organization of trade structures under imperfect infrastructure is of a special interest for emerging market economies. It is known that the trade sector plays a special part in an economy during transition to market economy. The trade sector adsorbs both significant capital investments and skilled human resources. Some economists believe that the accelerated development of the trade sector is favorable for the economy while some others argue that the trade is growing at the expense of producers distracting the economy's resources. Anyway, both acknowledge that the intermediaries are outdoing the rest in a decentralized transition economy. Probably, due to high transaction costs, poor infrastructure and high uncertainty, the trade profits are very high, and this attracts new resources and brings about further development of the trade sector. Note that unlike other transition economies, in Russian economy the private trade sector is being built from the scratch in the absence of developed trade infrastructure.

There is some literature on functioning and self-organization of trade networks. First, these are works [1]-[5] which examine decentralized trade systems when unlike Walrasian auction a number of local markets with different prices are assumed to exist. In spatial equilibrium models and other network equilibrium models [1],[2] the existence and properties of equilibrium are studied under different assumptions. Also, [1] suggests algorithms to compute network equilibria. [3],[4] consider price formation in pair-wise meetings of buyers and sellers with sequential bargaining. The convergence of such process to Walrasian equilibrium is studied. The paper [5] studies an exchange system with transaction costs which are described as time and number of computational steps required for trade. The paper suggests measurements for complexity in trade and examines the strategic role of debt-status and kinship in primitive exhange systems.

Second, in several works the agglomeration of trade middlemen has been considered. In [6] buyers and sellers change their location in order to maximize their objective functions. If the gain of mutual proximity is greater than the transportation costs, the agents form a market place. In [7] a population of several classes is considered and creation of local markets is studied. The formation of a local market in a given class incurs costs but without the local market the agents have to go to other classes' markets which is also unprofitable. There is shown that if transaction costs are low enough, there exist stable trade structures. And if the richest class' wealth far exceeds the others' then this class forms a monopolistic market. So [7] studies endogenous formation of trade networks, but it examines only final state of the system — ESS [8], like in network equilibrium models.

In the present work we apply a different methodology that allows to study both static and dynamical properties in the system. This approach referred to as *emergent computations* is currently widely used in economics, ecology, sociology etc. According to [9] and [10] the essence of this method is that information which is absent at lower levels can exist at the level of collective activities. Emergent computation approach can be applied to the collection of interacting agents, each following explicit instructions, also called "local programs", "micro-structure" and "component subsystems". The approach assumes that there is no central authority to control the system, the agents are able to adapt and learn, the global cooperation emerges from local interactions.

Interactions among the agents at a micro level bring about implicit global phenomena at the macroscopic level i.e. epiphenomena, for example, organization of stable structures in trade networks or existence of stable cycles in the dynamical behavior of the system. Therefore, the object of study is a nonlinear dynamical system which may evolve to stable cycles, strange attractors, chaotic behavior etc. as well as to a stable equilibrium. For systems which are complex enough it is often impossible to study all the dynamics arising analytically and computational experiments are used. This approach has proved to be effective in very many fields including modelling asset bubbles [11] and dynamical aspect of cooperative behavior [12]. In [13] it is applied to study a multi-sector and multi-stage production process with local interactions between productive units using non-convex technology. It has been proved that many small independent shocks to different sectors do not cancel in aggregate due to significantly non-linear interactions between units.

This approach is applied to study emergence and evolution of decentralized trade systems in [14] and [15]. In [14] the well-known evolutionary game approach is combined with endogenous partner selection. The optimal strategies for players evolve over time via a genetic algorithm, so agents are able to learn dynamically. In [15] a self-organized market with signalling costs is considered. The paper contains a specific model of bidding and signalling process. Like [14] learning is described with the aid of genetic algorithms combined with classifier systems. The results suggest that computational approach is useful to model decentralized trade systems.

The present work is similar to [14] and [15] as we also study self-organization of decentralized trade systems and apply similar methodology but we focus on characteristics of emerging trade networks in an economy with imperfect infrastructure and possibility of shortages.

In Section 2 we introduce the general setting and models of behavior of individual agents, in Section 3 results of analytical study and simulations are discussed. Section 4 contains some concluding remarks and directions of further studies.

2 The model

2.1 General setting

We consider interaction of economic agents of three types: consumers, producers and traders in a distributed market of homogeneous good. Each type is described by a set of parameters and rules of behavior in the market. Conventionally for emergent computations, we tend to simplify the rules of decision-making for individual agents,[2] and pay most attention to emergent properties of the whole system.

Denote sets of consumers, producers and traders by $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$, correspondingly. Assume that $\mathcal{P}$ and $\mathcal{T}$ are finite. Producers are pure sellers and their behavior is exogenous to the system. Consumers are pure buyers. Traders can either buy or sell. Buyers are indexed by $i \in \mathcal{C} \cup \mathcal{T}$, and sellers — by $j \in \mathcal{P} \cup \mathcal{T}$. Buyers can buy only one unit of good per transaction. For every pair (i, j) from $\mathcal{C} \cup \mathcal{T} \times \mathcal{P} \cup \mathcal{T}$ a nonnegative number r_{ij} is defined, which we will refer to as *trade distance*.[3] The trade distance is average time that buyer i has to spend to buy a unit of good from seller j. In an economy with developed infrastructure the time spent on purchasing is small and usually is not taken into account. However, in an economy with imperfect

[2] Firstly, this tendency corresponds to "bounded rationality" concept [16], and secondly, it is caused by requirement of minimum number of parameters in order to be able to comprehend the results of computer simulations.

[3] If we consider trade distances as an analogue of geographical distance, we should assume the triangle inequality $r_{ij} \leq r_{ik} + r_{kj}$ and the axiom of symmetry $r_{ij} = r_{ji}$. Below we will not require these axioms unless otherwise is specifically stated.

infrastructure such as contemporary Russian economy, the time for gathering current information about the seller, reaching the seller and physically transporting the good is significant, which, as shown below, may be crucial for macroscopic dynamical properties of trade networks.

Generally speaking, time spent on transaction is a realization of Poisson stochastic process with mean equal to trade distance; the processes are independent for different buyer-seller pair. However, simulations have proved that we may use mean flows of good instead of stochastic ones.

In addition to the time spent on purchasing we will also consider transportation costs as in spatial equilibrium models [1]. We assume that the transportation costs are proportional to the trade distance $K_{ij} = \kappa_i r_{ij}$, with coefficient κ_i being the cost of transportation per unit distance.

Every buyer has buying preferences α_{ij} i. e. if buyer i wants to buy a unit of good, he will go to seller j with probability α_{ij}. Naturally, we require $\alpha_{ij} \geq 0$, $\sum_j \alpha_{ij} = 1$ and $\alpha_{ii} = 0$, $i \in \mathcal{T}$. Therefore there is certain demand of buyer i for seller j's goods $\lambda_{ij} = \lambda^i \alpha_{ij}$, where λ^i - is total buyer i's demand.

Sellers are described by their selling prices p_j and probabilities of availability of the good β_j. We assume that sellers do not distinguish buyers so that any buyer that comes to seller j, buys a unit of good for p_j with probability β_j.

As it was mentioned above, the producers are passive suppliers of good. Their prices p_j and probabilities of absence of shortage β_j, $j \in \mathcal{P}$ are either constant parameters or exogenous functions of time in the model.

Thus the structure of the model is as follows.

1. Consumers: Each consumer as a buyer i knows all sellers' p_j and β_j (public information). Also knows their own characteristics: wage s_i, transportation cost κ_i,time for consumption of a unit of good τ_i, trade distances r_{ij} for all $j \in \mathcal{P} \cup \mathcal{T}$ (consumer i's private information). Make decisions on how much λ^i and from whom α_{ij} to buy determining $\lambda_{ij} = \lambda^i \alpha_{ij}$ and letting each seller know his λ_{ij}. Maximize consumption under budget constraint.

2. Traders: First, as a buyer i the trader knows all sellers' p_j and β_j (public information). Also he knows trade distances r_{ij} for all $j \in \mathcal{P} \cup \mathcal{T}$ (trader i's private information). The trader makes decisions on how much λ^i and from whom α_{ij} to buy determining $\lambda_{ij} = \lambda^i \alpha_{ij}$ and letting each

seller know his λ_{ij}. Second, as a seller j the trader knows present total demand for his goods $\lambda_i = \sum_{l \in C \cup T} \lambda_{ij}$. He makes decisions on sales volume choosing $\beta_j \in [0,1]$ and on selling price p_j and let know everyone $\beta_j p_j$. Maximize profits maintaining balance of sales and purchases.

Our model is a dynamical implementation of Bertrand competition [18] with trade distances. We believe that price adjustments are happening slower than the quantity adjustments. Also we believe that buying preferences are changing slower than quantities but faster than prices. Thus the hierarchy of time is as follows. In order to maintain balances (consumers' budget constraints and traders' material balances) at every moment of time we require immediate adjustment of flows of sales and purchases i.e. buyers' λ^i and sellers' β_i are set subject to the balance constraints under given prices and buying preferences. Then every buyer determine the buying preference profile α^*_{ij} which maximizes his objective function under given selling prices and slowly adjusts his current α_{ij} to the optimal point. Then sellers can observe buyers' response to the prices set and determine prices that maximize their objective functions and adjust the current prices p_i. We shall further refer to λ^i and β_i as *fast* variables and to α_{ij} and p_i as *slow* variables.

So the whole setting may be referred to as Bertrand-Nash one: the sellers set prices and observe buyers' response, change prices and observe response to new prices etc. As sellers do not cooperate, their attempts to maximize profit by changing price represent Nash-style tatonnement. Due to instantaneous adjustment of fast variables all required balances are maintained at every moment of time. In this framework, equilibrium prices are given by Nash equilibrium [19]. The corresponding game is defined in a normal form in Section 3.

We will not grant the agents with the memory of the past except trader who, in order to make sound pricing decision, should know demands for their goods λ_i at recent moments of time.

Let us now describe the behavior of individual consumers and traders.

2.2 Consumers

Consumer is described by a controllable Markovian process [20] with the following states: w — work, c — consumption, j — purchasing from seller j, $j \in \mathcal{P} \cup \mathcal{T}$. While working consumer receives wage s_i per unit of time.

He leaves the state w with Poisson rate Λ_i which is consumer's fast control variable — if consumer wants to increase working time he decreases the rate Λ_i.

Consumer selects a seller (state j) according to his buying preferences, i.e. with probability α_{ij}. The buying process is also a Poisson one with the rate $1/r_{ij}$ (so that average time spent in this state is r_{ij}). During the trip consumer spends κ_i dollars per unit of time. When the trip is over consumer either buys a unit of good at price p_j with probability β_j or buys nothing because of shortage with probability $1-\beta_j$. In the latter case he returns to the state w and resumes working, and in the former case he enters the state c and begins to consume the purchased unit.

The consumption process is also a Poisson one with the rate $1/\tau_i$ (i.e. average time of consumption of a unit of good equals τ_i). Variables $\tau_i \geq 0$, $s_i > 0$ and $\kappa_i \geq 0$ are individual characteristics of consumer.

We assume that consumer sets his fast variable Λ_i in order to maintain financial balance. In Appendix A it is shown that expected flow of consumption in this case is:[4]

$$U_i = \frac{\sum \alpha_{ij}\beta_j}{\sum \alpha_{ij}(\beta_j\tau_i + \beta_j p_j/s_i + r_{ij} + \kappa_i r_{ij}/s_i)} \quad . \tag{1}$$

In the Appendix B, we find α^*_{ij} that maximizes this functional over the simplex $\{\vec{\alpha}_i : \sum \alpha_{ij} = 1, \alpha_{ij} \geq 0\}$. It is shown there that the solution's j-th component is positive $\alpha^*_{ij} > 0$ if and only if

$$j \in J^* = \ \operatorname{Arg}\min_j \frac{p_j}{s_i + \kappa_i} + \frac{r_{ij}}{\beta_j} \quad .$$

In the generic case when $p_j/(s_i+\kappa_i) + r_{ij}/\beta_j$ are all different for different j, the consumer will tend to select only one seller: $\alpha^*_i = \vec{e}_{j^*}$, where $\vec{e}_{j^*}$ is j^*-th unit coordinate vector (so that $\alpha^*_{ij} = 0$ for $j \neq j^*$ and $\alpha^*_{ij^*} = 1$), and

$$j^* = \arg\min_j \frac{p_j}{s_i + \kappa_i} + \frac{r_{ij}}{\beta_j} \quad . \tag{2}$$

In the non-generic case, when there exist several such j^*, we will assume that the consumer shares his demand between these evenly $\vec{\alpha}^*_i = \sum_{J^*} \vec{e}_{j^*}/|J^*|$.

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[4]In this and further sections, the summation index is j if it is omitted: $j \in \mathcal{P} \cup \mathcal{T}$, $j \neq i$.

Figure 1: Buyer's choice of seller. The left graph illustrates the case of consumer and trader without shortage, $\operatorname{tg}\phi = s_i + \kappa_i$. The right one describes the case of trader with shortage.

The choice of j^* may be visualized as follows (Fig.1a). In the plane (q, p) we locate points (q_{ij}, p_j), where $q_{ij} = r_{ij}/\beta_j$, $j \in \mathcal{P} \cup \mathcal{T}$. Then we construct a Pareto-optimal part of the boundary of a convex hull $\mathcal{L}$ of the obtained set of points and draw a tangent line to this part of the boundary with slope equal to $s_i + \kappa_i$. The point of tangency will determine j^* (only in non-generic case here may be several points). One can see that if there is a distribution of consumers in the same location (so that r_{ij} are the same) with different wages s_i, then this distribution will be split into several segments with one seller from $\mathcal{L}$ serving consumers of one segment. Consumers with higher s_i will select seller with higher p_j and lower q_{ij}. If a seller decreases his price he widens his segment and increases his demand.

As the set of consumers is in general discrete, the variations of the traders' demand over time may be too large. In order to emulate continuity we consider the adaptation of the buying preferences with some finite rate rather than the instantaneous switching. In this case the agent adjusts his buying preferences trying to attain the desired ones, with the adjustment rate μ_i. So at every moment t current buying preferences $\alpha_{ij}(t)$ may be different from the desired ones $\alpha^*_{ij}(t)$:

$$\alpha_{ij}(t + \Delta) = \alpha_{ij}(t) + \mu_i \Delta(\alpha^*_{ij}(t) - \alpha_{ij}(t)) \ . \tag{3}$$

Here Δ is time step. We will assume Δ to be sufficiently small in comparison with $1/\mu_i$ so that $\mu_i \Delta \le 1$. The value $1/(\mu_i \Delta)$ shows the number of steps required for the consumer to adapt to the external changes.

For further study of traders' behavior we will also need a formula for average demand of i-th consumer for j-th seller's good per unit of time:

$$\lambda_{ij} = \alpha_{ij} / \sum \alpha_{ij} (\beta_j \tau_i + \beta_j p_j / s_i + r_{ij} + \kappa_i r_{ij} / s_i) \ . \tag{4}$$

2.3 Traders

As the traders can both buy and sell we should define two types of their behavior. First we shall consider the determination of fast variables under given slow ones then that of buying preferences and then that of the selling price.

Traders are also described by controllable Markovian processes. A trader can be either in a free state i, or in one of the states j of buying from seller j ($j \neq i$). In the free state, the trader can make a decision to buy a unit of good. In this case the trader leaves the state i and enters one of the states j with probability α_{ij}. The decision to leave the free state arrives with Poisson rate Λ_i. The rate Λ_i is the trader's fast variable which finally determines the flow of purchases.

As a seller, the trader receives the Poisson flow of buyers with the rate

$$\lambda_i = \sum_{k \in \mathcal{C} \cup \mathcal{T} \setminus \{i\}} \lambda_{ki} \ , \tag{5}$$

so that his average demand is λ_i units of good per unit of time. With probability β_i the trader sells his good at the price p_i to his buyers, and with probability $1 - \beta_i$ he refuses to sell. The trader may sell or refuse disregard to his current state. β_i is also the trader's fast variable and essentially determines the flow of sales.

Similar to consumer's, the process of buying is Poisson one with the rate $1/r_{ij}$; during the buying process trader spends κ_i amount of money every unit of time; when the process is over, he either buys a unit of good at price p_j with probability β_j or buys nothing with probability $1 - \beta_j$. In both cases the trader returns into the free state i and may go for the good again.

Unlike consumers who maintain the financial balance and maximize the inflow of good, traders tend to maintain the material balance (expected difference of sales and purchases is equal to zero) and to maximize profit (expected financial surplus per unit of time). In the Appendix C we show that the condition of material balance is equivalent to the following relationship between trader's fast variables Λ_i and β_i:

$$1/\Lambda_i = \sum \alpha_{ij}(\beta_j/(\beta_i\lambda_i) - r_{ij}) \ . \tag{6}$$

Note that $1/\Lambda_i$ must be nonnegative.

As shown in Appendix C the trader's expected profit equals

$$\Pi_i = \beta_i\lambda_i\left(p_i - \frac{\sum \alpha_{ij}(\beta_j p_j + \kappa_i r_{ij})}{\sum \alpha_{ij}\beta_j}\right) \ . \tag{7}$$

To find the fast controls first we shall maximize the functional (7) choosing $\beta_i \in [0,1]$ that satisfies the condition of non-negativity of (6).

We shall assume that the price p_i is such that the profitability condition holds:

$$p_i - \sum \alpha_{ij}(\beta_j p_j + \kappa r_{ij})/\sum \alpha_{ij}\beta_j \geq 0 \ . \tag{8}$$

Hence trader wants to increase β_i as much as allowed by conditions $\beta_i \leq 1$ and that of non-negativity of (6). There can be three cases.

1. *Shortage*. The demand is too high

$$\lambda_i > \frac{\sum \alpha_{ij}\beta_j}{\sum \alpha_{ij} r_{ij}} \ .$$

   In this case the trader can not serve all his demand and has to refuse to sell to some of his buyers $\beta < 1$. The share of non-satisfied demand is determined by the making (6) equal to zero (i.e. trader spends zero time in the state i):

$$\beta_i = \frac{1}{\lambda_i}\frac{\sum \alpha_{ij}\beta_j}{\sum \alpha_{ij} r_{ij}} < 1 \ , \quad 1/\Lambda_i = 0 \ .$$

2. *No shortage*. The demand is sufficiently low

$$\lambda_i < \frac{\sum \alpha_{ij}\beta_j}{\sum \alpha_{ij} r_{ij}} \ ,$$

   so that the trader can satisfy it completely, $\beta_i = 1$ and spends some time in the free state i:

$$\beta = 1 \ , \quad 1/\Lambda_i = \sum \alpha_{ij}(\beta_j/\lambda_i - r_{ij}) > 0 \ .$$

3. *Edge of shortage*. The demand is exactly equal to maximum possible supply under given slow variables:

$$\lambda_i = \frac{\sum \alpha_{ij}\beta_j}{\sum \alpha_{ij} r_{ij}} \ ,$$

so that both

$$\beta_i = 1 \ , \quad 1/\Lambda_i = 0 \ .$$

Although this case may seem non-generic, we will show below that buying preferences α_{ij} adjust so that it is as generic as the first two cases.

Thus the fast variables are determined. Now we are also able to calculate the demand of i-th trader for the good of the seller j per unit of time:

$$\lambda_{ij} = \alpha_{ij} \min\{q_i\lambda_i, 1\} / \sum \alpha_{ij} r_{ij} \ , \tag{9}$$

where

$$q_i = \left(\sum \alpha_{ij} r_{ij}\right) / \left(\sum \alpha_{ij}\beta_j\right) \ . \tag{10}$$

Now we shall describe the adjustment of the buying preferences to the desired ones. To find the latter the trader solves the problem of maximization of the functional

$$\Pi_i = \frac{\sum \alpha_{ij}(\beta_j(p_i - p_j) - \kappa_i r_{ij})}{\max\{\sum \alpha_{ij}\beta_j/\lambda_i, \sum \alpha_{ij} r_{ij}\}} \tag{11}$$

by choosing $\alpha_{ij} \geq 0$, $\sum \alpha_{ij} = 1$.

This optimization problem is solved in the Appendix D. The solution depends significantly on the magnitude of demand λ_i. If it is low enough then the trader satisfies all his demand $\beta_i = 1$ and chooses a seller that provides him with maximum profit per unit of good $j^* = \arg\max_j p_i - p_j - \kappa_i r_{ij}/\beta_j$. Graphically the choice of optimal seller j^* coincides with consumer's choice at $s_i \to 0$: in Fig.1a one has to draw the tangent line to the Pareto-optimal part of the boundary of the convex hull $\mathcal{L}$ with slope equal to κ_i. If $\kappa_i = 0$ then trader selects the seller with minimum selling price.

If trader's demand is too high then trader is bound to refuse to sell to some of his buyers $\beta_i < 1$ and chooses a seller that provides him with maximum profit per unit of time $j^* = \arg\max_j (p_i - p_j)\beta_j/r_{ij}$. Now the trader's choice depends on his own price p_i as well: one should draw a tangent line to $\mathcal{L}$

that goes through the point $(0, p_i)$ (Fig.1b). The more p_i is the more p_j and the less q_{ij} are. One can see that the trader's profit increases with slope $(p_i - p_j)/q_{ij}$ of the tangent line and therefore it increases with p_i since $\mathcal{L}$ is convex.

In the first case the trader will buy from a distant seller with the lowest selling price, in the second case — from some closer seller with a higher price. The intermediate situation is also possible in which the seller with the lowest price is too far away and buying only from him the trader would not be able to satisfy his demand, and vice versa the seller that provides maximum profit per unit of time is too close and buying from him the trader would be able to serve more buyers than he has. In this situation the trader diversifies his purchases and buys some amount from a remote seller at a lower price and the rest from a closer seller at a higher price in order to satisfy his demand exactly: $\beta_i = 1$ and $1/\Lambda_i = 0$. In the Appendix D it is shown that *all three situations are generic.*

Like those of consumer, current buying preferences of trader $\alpha_{ij}(t)$ may be different from the desired ones $\alpha^*_{ij}(t)$. In this case the trader adjusts his preferences to the desired ones with the adjustment rate μ_i according to the formula (3). Trader's adjustment rate μ_i is also assumed to satisfy $\mu_i\Delta \leq 1$.

To achieve higher profit, the trader can also change his selling price p_i. The price enters the expression (11) for profit both directly and indirectly as profit depends upon demand λ_i that, in turn, depends upon price according to (**??**), (5) and (9). If the trader knew all internal parameters of his buyers and had unlimited computation capacities, he would be able to calculate the dependence of $\lambda_i(p_i)$ exactly. However, a more realistic assumption is that the trader's capabilities to obtain, store and process information are limited, and in forecasting his demand function the trader uses only his observations of the demand in the past. We assume that after every adjustment of price the trader keeps the price constant for some time Δ_i and observes what happens. He believes that average demand per unit of time between two subsequent adjustments of price $\bar{\lambda}_i(t) = \Delta_i^{-1}\int_t^{t+\Delta_i}\lambda_i(\xi)d\xi$ is function of his price $p_i(t)$ during this period of time $[t, t+\Delta_i]$. Hence, using historical data on demand, the trader can estimate (locally) the derivative of demand by price

$$\frac{\partial\lambda_i}{\partial p_i}(t+\Delta_i) = F_i\left(p_i(t), \bar{\lambda}_i(t), p_i(t-\Delta_i), \bar{\lambda}_i(t-\Delta_i), ...\right) \ .$$

Here F_i is a function that gives forecast for demand sensitivity to price by past values of demand and price. E.g. $F_i = (p_i(t) - p_i(t-\Delta_i))/(\bar{\lambda}_i(t) - \bar{\lambda}_i(t-\Delta_i))$.

With current demand and sensitivity of demand given, the trader maximizes his profit function $\Pi_i(p_i)$ (11) over the interval $[(1 - \varepsilon_-^i \Delta_i)p_i(t), (1 + \varepsilon_+^i \Delta_i)p_i(t)]$. If he has shortage at this moment of time $\beta_i < 1$, he increases price to $(1 + \varepsilon_+^i \Delta_i)p_i(t)$, and if he has zero demand, he decreases price to $(1 - \varepsilon_-^i \Delta_i)p_i(t)$. In addition, the trader may not sell the good at a price below his marginal cost, so that his profit must be nonnegative and the profitability condition (8) must hold.

We require that the relative change of price be bounded by $-\varepsilon_-^i \Delta_i$ and $\varepsilon_+^i \Delta_i$ to provide the continuity of price over time at sufficiently small Δ_i (ε_-^i and ε_+^i are internal parameters of the i-th trader's). The matter is that trader's demand and therefore profit depend upon not only his behavior but also upon other traders'. This is why if we allow to change the price discontinuously in order to achieve the desired maximizer of profit instantaneously, the system would have oscillations of high magnitude or, in generic case, chaotic behavior caused by interrelationships and imperfect information of traders.

Also, to make the system more robust we let the moments for re-evaluation of price be stochastic rather than deterministic. We assume that re-evaluation moments arrive according to Poisson process with rate $1/\Delta_i$ (as in [21]). This removes artificial coordination of traders' decision-making and eliminates the oscillations that are caused by specifics of simulation methods rather than by the properties of the system itself.

Sellers' Δ_i should be greater than buyers' $1/\mu_j$ as in this case sellers really observe (at least partially) the impact of their price adjustment. If Δ_i are too small then sellers can not observe the actual demand function and therefore their price adjustments can only be based on the rule of thumb.

3 Dynamics of trade networks

3.1 Equilibria and oscillations

So we have defined the dynamical system, the current state in which is given by the set of the slow variables for all agents: α_{ij}, p_j. In this section we shall study the properties of the whole trade network. First, we shall look for equilibria. We will consider the state of the system to be an equilibrium if all buying preferences are optimal and all traders' prices are local maximizers of their profit functions. The profit functions $\Pi_i(p_i)$ are obtained under given other sellers' prices from (11) with demands (5), (4) and (9), and buying preferences. Note that by definition, in equilibrium $\beta_i = 1$ for all $i \in \mathcal{T}$, as if a trader has shortage, his profit function $\Pi_i(p_i)$ is increasing. We need to exclude cases when traders have no demand and are able to lower prices to attract some buyer. Therefore we'll define profit function $\Pi_i(p_i) = -\infty$ if $\lambda_i(p_i) = 0$.

Assume that the wages and trade distances are bounded and distribution of consumers in space and by wages is such that every trader's profit function is concave for all prices of other traders given. Then a state is an equilibrium if and only if it is a Nash equilibrium in the following game: the set of players is $\mathcal{T}$, their strategies are prices p_i (under the assumptions made p_i belong to certain compact sets) and their payoff functions are the profit functions $\Pi_i(p_i)$ defined above. In this game Nash equilibrium exists.

All the assumptions are natural except concavity of profit functions. If a trader i has shortage $\beta_i < 1$, his profit function $\Pi_i(p_i)$ is increasing and concave. But if a trader has no shortage $\beta_i = 1$ it depends upon distribution of consumers. In this case the profit received from selling to a single consumer is an increasing fraction-linear function until the consumer moves to another seller and profit received from him discontinuously falls down to zero. The trader makes more profit on consumers who still buy from him but he loses all profit from the consumers gone. If distribution of consumers is uniform (approximately same number of consumers at every level of wage in every point of the metric space) then with increase of price the profit first grows slowly and then falls slowly and may be concave. But if the distribution is clustered like in the case corresponding to Fig.2 then after the trader loses a whole cluster of consumers due to infinitesimal price increase, his profit falls by finite quantity as increase in profit from remaining customers is infinitesimal. Further price increase contributes continuous increase in profit until the trader loses next cluster. In this case the profit function has several local maxima and is not concave so that the Nash equilibrium may fail to exist. However, the equilibrium in the dynamical system considered may still exist in the absence of Nash equilibrium, moreover, there may be several equilibria.

The consequence of this fact is that the average trading profit over all traders may appear to be non-smooth function of time, as we can see in the

Figure 2: Profit function in case of clustered distribution of consumers. The trader buy the good at producer's price.

Fig.3. In this case initially traders were in some local optima. Because of consumers' distribution that leads to multiple local optima it is possible for system to switch between them rather abruptly. It is important to emphasize that such jump of average profit does not correspond to any jump of average price. The temporary change in profit is just a consequence of redistribution of consumers' demand among sellers. In the case illustrated on the Fig.3 the higher maximum appears to be less stable than the lower one so the system finally switches to the previous one.

This helps to explain persistent oscillations that occur in the system. As the local maxima of the individual's profit function depend upon other traders' prices, the change of price of trader i may make trader j switch from seeking one local maximum to another one, consequently change of price of trader j will influence profit function of i and make the latter to change his price again etc.

The other source of instability contributed by singularities in distribution of consumers is caused by sudden shortages. This danger is significant when consumers change their buying preferences too fast (high μ_i). If a large group of consumers has same location and wage then a small change of trader j_1's price may make them go to another trader j_1. If the group is large enough, trader j_2 that used to have no shortage before, will have shortage now, so his attractiveness to consumers will fall abruptly by finite quantity. Then

Figure 3: Switching between two different local optima. The graph shows evolution of aggregate traders' profit over time.

the whole group of consumers will go back to trader j_1 and create shortage there etc. Note that if consumers'adaptation rates μ_i are small enough the shortage occurred will not be large and the trader will have time to overcome it by increasing his price so the trader perceives his profit function to be continuous. The situation becomes more critical with the worsening of the infrastructure as the small changes in prices now generate relatively high levels of shortage.

3.2 Impact of imperfect infrastructure

The average time spent on buying a unit of good q_i is very important for both consumers and traders. The expression (10) for q_i contains both β_j that are determined as a result of interaction of agents and trade distances r_{ij} that are parameters. The greater r_{ij} are, the more time buyers spend on buying, so it is reasonable to consider r_{ij} as a measure of imperfection of infrastructure. In order to study impact of imperfection of infrastructure, we will compare systems in which all trade distances differ by ρ times, i.e. we suppose that the trade distance matrix is proportional to some given matrix $r_{ij} = \rho R_{ij}$ and will study dependence of dynamical properties on the factor ρ with all other parameters fixed, except for $\kappa_i \sim K_i/\rho$. The latter is required to distinguish the phenomena caused by imperfect infrastructure from ones

Figure 4: Convergence to equilibrium under near-perfect infrastructure. The graph shows evolution of average traders' price over time. All producers' prices are equal to 1.

caused by increase in transportation costs.

If $\rho \to 0$, then there is near perfect infrastructure, price differentials and trade profits are also small and there are no shortages. Indeed, if $\sup s_i < \infty$ then demand (5) is bounded although increasing at $\rho \to 0$, therefore β_j are at least separated from zero and $q_i \to 0$. Hence, $q_i\lambda_i \to 0$, and there are no shortages $\beta_i = 1$. In this case system quickly converges to equilibrium without oscillations (Fig.4).

Proposition 1. Assume the following

- triangle inequality holds $r_{ij} \leq r_{ik} + r_{kj}$ for all i, j, k,
- all agents' transportation costs are the same $\kappa_i = \kappa$ for all i,
- producers do not have shortage $\beta_j = 1$, $j \in \mathcal{P}$.

Then after adaptation of buying preferences (i.e. $\alpha_{kl} = \alpha^*_{kl}$ for all k, l) trader i will buy from another trader only if $q_i\lambda_i \geq 1$.

Proof. Let the opposite be the case. Suppose that $q_i\lambda_i < 1$ and the trader i wants to buy from another trader j. As every trader maintains balance of sales and purchases there must exist a chain of sellers with one (and only one) producer through which the goods bought by i come. Without loss of generality renumerate the traders so that $i > j$ and the chain is $j, j-1, ..., 1, 0$ with 0 being the producer. Then using the Proposition in the end of Appendix 4 we may write that

$$p_j > p_{j-1} + \kappa r_{j,j-1}/\beta_{j-1} > ... > p_0 + \kappa \sum_{k=1}^{j} r_{k,k-1}/\beta_{k-1} \geq p_0 + \kappa \sum_{k=1}^{j} r_{k,k-1}$$

The triangle inequality implies $r_{io} \leq r_{ij} + \sum_{k=1}^{j} r_{k,k-1}$. Therefore $p_0 + \kappa r_{i0} < p_j + r_{ij} \leq p_j + \kappa r_{ij}/\beta_j$.

Introducing $q_i\lambda_i < 1$ into (11) we see that the trader should maximize $p_i - (p_l + \kappa r_{il}/\beta_l)$. Then he should certainly prefer buying from producer 0 directly to buying from trader j (and therefore through the whole chain$j, j-1, ..., 1, 0$).

According to Proposition 1, if ρ is sufficiently small, either consumer will buy directly from producer or consumer will buy from a trader who will buy from producer, and there can not be any chain of traders serving consumers. To prove this proposition let us consider the middleman's profit: assume that the trader 1 buys from the producer 0 and sells to some other trader 2. Non-negativity of the profit implies that $p_1 > p_0 + \kappa_1 r_{10}$. Assuming that $\kappa_1 = \kappa_0$ we can obtain from triangle inequality: $p_1 + \kappa_1 r_{21} > p_0 + \kappa_2 r_{20}$ which implies that the trader 2 would prefer the producer rather than the trader 1. If first two assumptions of Proposition 1 are violated and some traders possess better technologies, the resulting economy of scale may encourage emergence of trade hierarchies. Although as economy of scale in this cases is due to exogenous definitions rather than the self-organization processes we will further study emergence of trade structures under the assumptions of Proposition 1.

When ρ increases, traders' prices grow no faster than linearly with ρ. Indeed, for any j if $p_j > p_k + (s_i + \kappa_i)(r_{ij} - r_{ik})$, $k \in \mathcal{P}$ then consumer i will buy from producer k. If $\sup s_i < \infty$ then traders will lose all their demand when prices grow faster than linearly with ρ. Therefore in case of non-trivial trade network prices grow not faster than linearly with ρ and consumers' demand (**??**) falls as $a/(\rho + b)$. Hence quantities $q_i\lambda_i \sim a\rho/(\rho + b)$ increase with ρ. The coefficients a, b are determined by relative trade distances R_{ij} (the network structure), real wages s_i/p_j, $i \in \mathcal{C}$, $j \in \mathcal{P}$ and the consumption rate $1/\tau_i$.

Figure 5: Phase transition because of worsening infrastructure. The graph shows dependence of all traders' profit Π on imperfection of infrastructure ρ in logarithmic scale.

Thus, deterioration of infrastructure results in qualitative change in the self-organization processes. We can see that the trade profit depends on ρ non-smoothly.

When ρ is small all traders have no shortage and no long chains of traders exist. The system quickly converges to near-perfect equilibrium and traders' profits are proportional to ρ (see $45°$ slope in the left part of the graph) i.e. small. But when ρ grows large enough the phase transition takes place: as quantities $q_i\lambda_i$ grow the shortages become more likely. There is no shortage in equilibrium, but one should distinguish equilibria with $\beta_i = 1$, $1/\Lambda_i > 0$ and $\beta_i = 1$, $1/\Lambda_i = 0$. The former is more likely to happen at smaller ρ and the latter is more typical for greater ρ. The latter corresponds to the case when a trader buys from two sellers; unlike the former, it is an equilibrium at the edge of shortage. A small change of other traders' behavior may make the trader fall into shortage. One should mention that once caught in the shortage trap, the system can not rapidly get out: every trader tries to get rid of shortage but if all his counterparts have shortages and producers are too far, he simply does not have time to satisfy all his demand. So what happens is the bursts of shortages that traders slowly take over. But then seeking for equilibrium, which for many traders is likely to be reached at $q_i\lambda_i = 1$, the traders generate another burst etc. (Fig.6). Note that in the

Figure 6: Bursts of shortages at the critical value of parameter ρ. The graph shows time series of average $1-\beta_i$, $i \in \mathcal{T}$.

phase transition $q_i \lambda_i \geq 1$ so that traders buy from each other and long chains of traders do exist. This leads to an abrupt increase in trade profits as can be seen in Fig.5. Note that in this case traders buy from each other without dropping assumptions of Proposition 1. Trade hierarchies emerge due to imperfect infrastructure rather than due to economy of scale that wholesale traders possess if the triangle inequality is violated or they have lower κ_i. The effects of clusters and imperfect information become more significant while the degree of imperfection of the infrastructure increases.

Further worsening of infrastructure gives rise to inability of traders to serve all consumers, so consumers with low s_i (poor consumers) will prefer to buy directly from producers,[5] and traders can only satisfy the demand of upper segment of consumers that buy at prices much higher than original prices of producers. Therefore in some cases the trading profit may even decrease with ρ.

[5] This is what happens in Russia. Many individuals are going abroad for shopping. There is an estimate of USD 11 bln. of consumer goods imported annually by individuals which accounts for tens percents of overall Russian import.

Figure 7: A multi-level structure. Trade distances are equal to Euclidean 2-D distances between agents in the diagram.

3.3 Emergence of trade structures and types of behavior

Thus the structure of trade links depends significantly on the degree of imperfection of the infrastructure. If the infrastructure is near perfect, all traders are far from shortage, and stable structures are as follows: some consumers buy from traders that buy directly from producers, other consumer that are closer to producers buy from producers themselves, some traders do not sell nor buy at all. In case of supercritical infrastructure, most traders are at the edge of shortage, so the stable structure do include long chains and hierarchies of traders, some consumers buy directly from producers.

When the infrastructure is critical, stable structures are not observed. Instead, long chains of traders emerge and then disappear spontaneously, even with reversing directions of good flows between traders.

The two most interesting types of behavior emerging may be referred to as stabilizing hierarchy and destabilizing speculation. The former corresponds to critical or supercritical values of ρ (i. e. after the phase transition), the latter is typical for the critical situation. Both types of roles require an appropriate geometrical layout of the system.

The former case corresponds to the multi-level system as in Fig.7. Consumers are located at the lowest level D. They buy from the sellers at the

Figure 8: Price destabilizing speculator: a fragment of long-run price oscillations. The graph shows time series of the speculator's price (bold line, left scale) and average price of other traders (right scale). Average time of price re-evaluation is 5 units of time.

level C while they prefer to buy from the wholesale seller B because of having shortage. The seller B is buying from the producer A. For μ_i sufficiently high it will be a persistent oscillations in the demand for the traders' C1 and C2 good, as we have shown above. However, the demand for the seller B's good is not oscillating, or at least is not oscillating as much as the demand at the lower levels, because at every moment of time either C1 or C2 has shortage so he is likely to buy from B. On the other hand if there were no oscillations at the lower level, trader B would have no demand from the C level and therefore no profit (this is observed when infrastructure is too good $\rho \to 0$). Thus one can say, that emergence of wholesale trader is observed, the type of trader that makes profit out of stabilizing lower level oscillations.

The other case requires the distribution of consumers to be singular and an inefficient trader in the system to present. This trader potentially can't obtain non-zero profit in the static case, i.e. it takes for him too much time to serve even minimum value of the possible demand (note that it is discrete). Initially this trader can't help to lose all his buyers . Then, according to the rules we have set, he is decreasing his price in order to have non-zero demand. After he gains some buyers his demand is increasing because of low price and not very high level of shortage. Eventually his attractiveness for buyers falls because a) the level of shortage becomes to be too bad and b) his price is too high as he is forced to increase his price in order to overcome it. He loses all the demand he have initially obtained. All this lost demand now have to be distributed among the rest of the traders. As they have already adjusted to lower demand they face sudden shortage. Then the inefficient trader begins to decrease price in order to gain some buyers etc.

So the inefficient trader generates long-run oscillations in the system. It is worth while noting that the period of the oscillations is much greater than the traders' planning and observation horizons. However, the inefficient trader who generates the oscillations actually benefits from the oscillation; his average profit for the period may be even higher than that of more efficient traders. Therefore we can say that his behavior is rational in the long-run, although his instructions are set up only for short-term.

4 Concluding remarks

We have considered a model that can be applied for study of self-organization processes in trade networks in an economy with imperfect infrastructure. Such model may be applied for analysis of market for imported consumer goods in Russia with international suppliers being denoted as producers. The behavior of world market does not depend upon processes in Russian economy, so producers' prices are exogenous. In addition to analysis of self-organization processes, the model can be used for analysis of the trade networks' response to exchange rate shocks or introduction of new import tariffs etc. Also, the model can be applied to analysis of development of trade in Europe before the Industrial Revolution, as the model provides a study of trade structure that emerge under different degrees of infrastructure development.

The main result of both analytical and computational investigation of the model is that evolution of trade networks does depend significantly on the degree of imperfection of infrastructure ρ. In the case of near-perfect infrastructure trade network quickly converges to near-competitive equilibrium which does not differ qualitatively from equilibrium with perfect infrastructure. However, if infrastructure is worsening a phase transition takes place: long chains of traders emerge, their profits increase considerably, regular bursts of shortages can be observed. After critical value of parameter ρ the traders lose poorer part of consumers who start buying directly from producers. The factors that influence stability of the system are heterogeneity of consumers, agents' adaptation rates and geometry of the trade network. Note that the quick convergence to an efficient equilibrium at $\rho \to 0$ proves adequacy of agents' decision-making procedures for an economy with near-perfect infrastructure, although these procedures are far less successful under more imperfect infrastructure.

Computer simulations have shown existence of multiple equilibria including inefficient ones. In further works we will study factors that determine efficiency and stability of final equilibrium states or limit cycles. We also plan to enhance the model by introducing natural selection of traders with different (and evolving over time due to learning) forecasting and pricing procedures, or by giving traders possibilities to change their location i.e. r_{ij} and r_{ki}. There may also be worth while considering mechanisms of cooperation for coordinating traders' pricing policies or for investment of part of profit into infrastructure.

All phenomena characteristic for critical and supercritical state of the system have been observed in Russian economy since 1992. The results of the paper suggest that development of infrastructure can dramaticly reduce trade profits and therefore improve consumer welfare.

5 Appendices

5.1 Consumer's fast controls

Leaving out index i, denote by P_w, P_c, P_j the probability distribution in the steady state of the controllable Markovian process. To calculate these, we write equations of balance of probability inflows and outflows in every state:

$$\begin{aligned} P_c/\tau_i &= \sum P_j \beta_j / r_{ij} \ , \\ P_w \Lambda_i &= P_c/\tau_i + \sum P_j (1-\beta_j)/r_{ij} \ , \\ P_j/r_{ij} &= P_w \Lambda_i \alpha_{ij} \ . \end{aligned}$$

By definition, $P_w + P_c + \sum P_j = 1$. We solve this system of equations and find

$$\begin{aligned} P_w &= \{1 + \Lambda_i \sum \alpha_{ij}(r_{ij} + \tau_i \beta_j)\}^{-1} \ , \\ P_c &= \{1 + \Lambda_i \sum \alpha_{ij}(r_{ij} + \tau_i \beta_j)\}^{-1} \Lambda_i \tau_i \sum \alpha_{ij} \beta_j \ , \\ P_j &= \{1 + \Lambda_i \sum \alpha_{ij}(r_{ij} + \tau_i \beta_j)\}^{-1} \Lambda_i \alpha_{ij} r_{ij} \ . \end{aligned}$$

Now we write expected change of the consumer's wealth over time (the difference between consumer's income and expenses):

$$P_w s_i - P_w \Lambda_i \sum \alpha_{ij}\beta_j p_j - \kappa_i \sum P_j \ .$$

In order to make the above equal to zero and keep financial balance, the consumer sets his fast variable Λ_i that determines how much time to work and therefore how much money to spend on consumption:

$$\Lambda_i = \left\{\sum \alpha_{ij}(\beta_j p_j + \kappa_i r_{ij})/s_i\right\}^{-1} \ .$$

Then we can find expected flow of consumption in the steady state:

$$U_i = P_c/\tau_i = \frac{\sum \alpha_{ij}\beta_j}{\sum \alpha_{ij}(\beta_j \tau_i + \beta_j p_j/s_i + r_{ij} + \kappa_i r_{ij}/s_i)} \ .$$

5.2 Maximization of fraction-linear functional over simplex

Lemma. Let J be a finite set, and $B_j \geq 0$ for all $j \in J$, while $\sum_{j\in J} B_j^2 > 0$. Then the solution of the problem of maximization of the functional

$$\sum_{j\in J} \alpha_j A_j / \sum_{j\in J} \alpha_j B_j \tag{2}$$

over simplex $\mathcal{A} = \{\vec{\alpha} : \alpha_j \geq 0, \sum_{j\in J} \alpha_j = 1\}$ is a set of all $\vec{\alpha} \in \mathcal{A}$ such that $\alpha_j = 0$ if $A_j/B_j < \bar{A} = \max\limits_{k\in J} A_k/B_k$.

Proof. Denote $J^* = \operatorname{Arg}\max\limits_{k\in J} A_k/B_k$. For all $\vec{\alpha}$ such that $\alpha_j = 0$ for $j \notin J^*$, the functional's value is equal to $\bar{A}$, since in this case

$$\frac{\sum\limits_{j\in J} \alpha_j A_j}{\sum\limits_{j\in J} \alpha_j B_j} = \frac{\sum\limits_{j\in J^*} \alpha_j A_j}{\sum\limits_{j\in J^*} \alpha_j B_j} = \frac{\bar{A} \sum\limits_{j\in J^*} \alpha_j B_j}{\sum\limits_{j\in J^*} \alpha_j B_j} = \bar{A} \ .$$

For any other $\vec{\alpha}$ the value of the functional is less then $\bar{A}$, because

$$\frac{\sum\limits_{j\in J} \alpha_j A_j}{\sum\limits_{j\in J} \alpha_j B_j} = \frac{\sum\limits_{j\notin J^*} \alpha_j A_j + \bar{A} \sum\limits_{j\in J^*} \alpha_j B_j}{\sum\limits_{j\notin J^*} \alpha_j B_j + \sum\limits_{j\in J^*} \alpha_j B_j} < \frac{\bar{A} \sum\limits_{j\in J} \alpha_j B_j}{\sum\limits_{j\in J} \alpha_j B_j} = \bar{A} \ .$$

5.3 Producer's fast controls

Denote by P_i, P_j probability distribution in the steady state of the Markovian process. Then we write the balance of probability inflows and outflows in every state:

$$\begin{aligned} P_i\Lambda_i &= \sum P_j/r_{ij} \ , \\ P_j/r_{ij} &= P_i\Lambda_i\alpha_{ij} \ . \end{aligned}$$

Using $P_i + \sum P_j = 1$ we find

$$\begin{aligned} P_i &= \{1+\Lambda_i \sum \alpha_{ij} r_{ij}\}^{-1} \ , \\ P_j &= \{1+\Lambda_i \sum \alpha_{ij} r_{ij}\}^{-1} \Lambda_i \alpha_{ij} r_{ij} \ . \end{aligned}$$

Let us now write the condition of material balance. Expected change of trader's stock of the good equals to the difference of purchases and sales per unit of time, that is:

$$P_i\Lambda_i \sum \alpha_{ij}\beta_j - \beta_i\lambda_i \ .$$

Substituting for P_i and making equal to zero we obtain the relationship between trader's fast variables Λ_i and β_i:

$$1/\Lambda_i = \sum \alpha_{ij}(\beta_j/(\beta_i\lambda_i) - r_{ij}) \ .$$

Now we shall find the trader's expected profit. In the steady state the net cash flow equals to

$$\Pi_i = p_i\beta_i\lambda_i - P_i\Lambda_i \sum \alpha_{ij}\beta_j p_j - P_j\kappa_i = \beta_i\lambda_i\left(p_i - \frac{\sum \alpha_{ij}(\beta_j p_j + \kappa_i r_{ij})}{\sum \alpha_{ij}\beta_j}\right) \ .$$

5.4 Maximization of (11) over simplex

Let us denote $A_j = \beta_j(p_i - p_j) - \kappa_i r_{ij}$, $B_j = \beta_j/\lambda_i$, $C_j = r_{ij}$. Then the functional (11) turns to

$$\frac{\sum \alpha_{ij} A_j}{\max\{\sum \alpha_{ij} B_j, \sum \alpha_{ij} C_j\}} \ ,$$

where $B_j > 0$ and $C_j > 0$ for all j. To solve the problem of maximization of this functional over the simplex $\mathcal{A} = \{\vec{\alpha}_i : \alpha_{ij} \geq 0, \sum \alpha_{ij} = 1\}$ we should consider three cases.

1. There exists k such that $B_k \geq C_k$ and $A_k/B_k \geq A_j/B_j$ for all j. Then the solution is $\vec{\alpha}_i^* = \vec{e}_k$. If there are several such k, then any convex combination of vectors $\vec{e}_k$ is a solution, as for any convex combination the value of functional is the same and equal to A_k/B_k. For any other vector $\vec{\alpha}_i$ we have

$$\frac{\sum \alpha_{ij} A_j}{\max\{\sum \alpha_{ij} B_j, \sum \alpha_{ij} C_j\}} \leq \frac{A_k}{B_k} \frac{1}{\max\{1, (\sum \alpha_{ij} C_j)/(\sum \alpha_{ij} B_j)\}} \leq \frac{A_k}{B_k} \ .$$

   This first case corresponds to situation when the trader can satisfy all his demand. Substituting for A_j and B_j we find that in this case the trader selects a seller to buy from in order to maximize profit per unit of good:

$$k \in \ \operatorname{Arg}\max_j (p_i - p_j - \kappa_i r_{ij}/\beta_j) \ .$$

2. There exists k such that $B_k < C_k$ and $A_k/C_k \geq A_j/C_j$ for all j. Then the solution is $\vec{\alpha}_i^* = \vec{e}_k$. If there exist several such k then any convex combination of vectors $\vec{e}_k$ is a solution. The proof is similar to the above.

   This case corresponds to the situation when the demand λ_i is, on the contrary, too large, and the trader has shortage: $\beta_i < 1$. Substituting for A_j and C_j we find that in this case the trader selects a seller to buy from in order to maximize profit per unit of time. Now, even in the absence of transportation costs the trader takes into account seller's shortage and the distance to the seller:

$$k \in \ \operatorname{Arg}\max_j \left((p_i - p_j)\beta_j/r_{ij} - \kappa_i\right) \ .$$

   It is easy to show that conditions of the cases 1 and 2 are inconsistent. Indeed, let k_1 and k_2 exist, that meet the conditions of the cases 1 and 2, correspondingly, then $B_{k_1} \geq C_{k_1}$, $B_{k_2} < C_{k_2}$, $B_{k_1}/B_{k_2} \leq A_{k_1}/A_{k_2} \leq C_{k_1}/C_{k_2}$. These inequalities are evidently inconsistent.

3. Thus cases 1 and 2 exclude each other but they do not cover all the set. If neither 1 nor 2 is the case then we claim that the solution lies in hyper-plane $\Gamma = \{\vec{\alpha}_i : \sum \alpha_{ij} B_j = \sum \alpha_{ij} C_j\}$, and in generic case the solution is a convex combination $x_{kl}\vec{e}_k + (1 - x_{kl}\vec{e}_l)$ of *two* unit

vectors $\vec{e}_k$ and $\vec{e}_l$ that lie in the opposite half-spaces of hyper-plane Γ: $B_k - C_k \geq 0$, $B_l - C_l < 0$. As the solution belongs to the simplex, x_{kl} can be found easily:

$$x_{kl} = \frac{C_l - B_l}{B_k - C_k + C_l - B_l} \ , \tag{3}$$

and the value of the objective function equals

$$\Pi_{kl} = \frac{A_k(C_l - B_l) + A_l(B_k - C_k)}{B_k C_l - B_l C_k} \ . \tag{4}$$

To find the real solution one should look up all the pairs of vectors $\vec{e}_k$ and $\vec{e}_l$ that lie in the opposite half-spaces of Γ and find the pair k^*, l^* that maximizes Π_{kl}.

Now we will prove that $\vec{\alpha}_i^* = x_{k^*l^*}\vec{e}_{k^*} + (1 - x_{k^*l^*})\vec{e}_{l^*}$ given by this procedure is the solution sought. First, we shall show that the solution must lie in Γ, i.e. $\sum \alpha_{ij}^* B_j = \sum \alpha_{ij}^* C_j$. For any vector $\vec{\alpha}_i$ such that $\sum \alpha_{ij} B_j > \sum \alpha_{ij} C_j$ consider a convex combination $\vec{\alpha} = \xi\vec{\alpha}_i + (1-\xi)\vec{e}_{j_B}$, where $j_B \in \operatorname{Arg\,max} A_j/B_j$. By definition of the third case $B_{j_B} < C_{j_B}$, i.e. $\vec{\alpha}_i$ and $\vec{e}_{j_B}$ lie in the opposite half-spaces of Γ, and there exists $\xi \in (0,1)$ such that $\vec{\alpha}_i \in \Gamma$. One can easily see that the value of functional is greater at $\vec{\alpha}_i$ than at $\vec{\alpha}_i$. Hence the situation $\sum \alpha_{ij}^* B_j > \sum \alpha_{ij}^* C_j$ is not possible.

Similarly, considering the convex combination with the vector $\vec{e}_{j_C}$, where $j_C \in \operatorname{Arg\,max} A_j/C_j$, we can show that the case $\sum \alpha_{ij}^* B_j < \sum \alpha_{ij}^* C_j$ should be excluded as well. The only remaining possibility is $\sum \alpha_{ij}^* B_j = \sum \alpha_{ij}^* C_j$, i.e. $\alpha^* \in \Gamma \cap \mathcal{A}$. $\Gamma \cap \mathcal{A}$ is a convex polyhedron with extreme points $x_{kl}\vec{e}_k + (1 - x_{kl})\vec{e}_l$, where $k \in K = \{j : B_j \geq C_j\}$, $l \in L = \{j : B_j < C_j\}$, and x_{kl} is given by (3). Every point $\vec{\alpha}$ of a convex polyhedron may be represented as a convex combination of its extreme points so that there are $a_{kl} \geq 0$, $\sum_{k\in K}\sum_{l\in L} a_{kl} = 1$ such that:

$$\vec{\alpha} = \sum_{k\in K}\sum_{l\in L} a_{kl}(x_{kl}\vec{e}_k + (1 - x_{kl})\vec{e}_l) \ .$$

Substituting into the functional we find that the maximum is reached when and only when $a_{kl} = 0$ if $\Pi_{kl} < \Pi_{k^*l^*}$. In generic case when all Π_{kl} are different, the maximum is reached at $a_{k^*l^*} = 1$. If there are several pairs (k^*, l^*) such that $\Pi_{k^*l^*} = \max_{kl} \Pi_{kl}$, any convex combination of vectors $x_{k^*l^*}\vec{e}_{k^*} + (1 - x_{k^*l^*})\vec{e}_{l^*}$ is a solution.

Using the formula 4 we can prove the following important

Proposition. Assume that there exist such j that $A_j > 0$. Then the following is true: if $\alpha^*_k > 0$ then $A_k > 0$.

Proof. If the maximum falls into cases 1 or 2 the statement is quite obvious: $\alpha^*_k = e_{k^*}$ and by definition the case $A_{k^*} \leq 0 < A_j$ is impossible.

If case 3 is the case, the maximum will be reached only on such k and l that $A_k > 0$ and $A_l > 0$. Assume the opposite. Let $A_k \leq 0$. Then using $C_l > B_l$ we obtain

$$\Pi_{kl} \leq A_l \frac{B_k - C_k}{B_k C_l - B_l C_k} = \frac{A_l}{C_l} \frac{B_k C_l - C_l C_k}{B_k C_l - B_l C_k} < \frac{A_l}{\max\{B_l, C_l\}}$$

This means $\vec{\alpha} = e_l$ provides better value of functional than $x_{kl}\vec{e}_k + (1 - x_{kl}\vec{e}_l)$.

This proposition actually says that the trader i will never tend to buy from a seller j with $p_j + \kappa_i r_{ij}/\beta_j \leq p_i$ if there exists at least one seller with $p_j + \kappa_i r_{ij}/\beta_j < p_i$. If the latter seller does not exist then profitability condition (8) requires that under trader i should not buy (and therefore sell) anything at all.

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