Chapter 1. Introduction of the Feasible Goals Method

In this Chapter the Feasible Goals Method (FGM) is introduced. A simple regional environmental model is considered. The FGM is used for supporting the search for environmentally sound strategies of regional development. Then, the Interactive Decision Maps (IDM) visualization technique is introduced. Thereafter, FGM/IDM technique is discussed, and a demo Web resource is described that illustrates application of the FGM on Internet for the development of independent environmental strategies. Finally, the mathematical formulation of the FGM/IDM technique is given, and its algorithmic basis is discussed.

1.1. The FGM and its application in a regional environmental problem

We introduce the FGM on the basis of a simple water-related model of regional production. The original version of the model was elaborated in the beginning of 80s at the International Institute for Applied Systems Analysis (IIASA), Austria. The model that described water-related problems of the South-Western Skane region of Sweden was explored using the FGM (Bushenkov et al., 1982a). Later, the model was modified to make the conflict between production and environment more acute, and so now it is far away from the original problem. The modified model was used for introduction of the FGM in (Lotov, 1984), and since then it is permanently used for this purpose. It is applied now in computer laboratory works in several universities in Russia and abroad. An educational environmental computer game called LOTOV_LAKE

(Lotov et al., 1992) and a demo Web resource considered in this Chapter are based on exploration of the same model, too.

Regional environmental problem

A region with intensive agricultural production is considered. The region is located in the basin of a river that runs through a lake and then flows into sea (Figure 1.1.1). The lake serves as the municipal water supply and is an important environmental and recreational site.

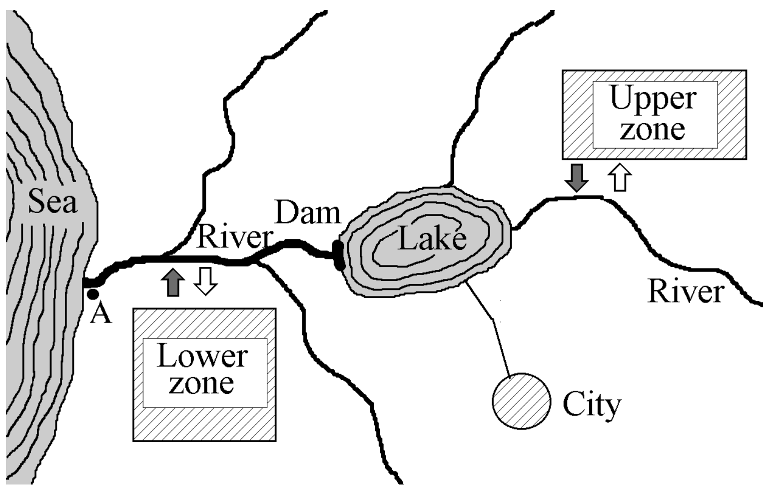

Figure 1.1.1. Map of the region

The problem of economic development of the region is studied. If the agricultural (to be precise, grain-crops) production would increase, it may spoil the environmental situation in the region. This is related to the fact that the increment in the grain-crops output requires irrigation and application of chemical fertilizers. It may result in negative environmental consequences, namely, a part of the fertilizers may find its way into the river and the lake with the withdrawal of water. Moreover, shortage of water in the lake may occur during a season.

Two agricultural zones are located in the region. Irrigation and fertilizer application in the upper zone (located higher than the lake) may result in a drop of the level of the lake and in the increment in water pollution. Irrigation and fertilizer application in the second zone that is located lower than the lake may also influence the lake. This influence is, however, not direct: irrigation and fertilizer application in the lower zone may require additional water release from the lake into the river (the release is regulated by a dam) to fulfill the requirements of pollution control at the monitoring station located in point ***A*** (Figure 1.1.1).

A finite number of grain-crop production technologies are considered in the model. Intensive technologies are related to high levels of water consumption and fertilizer application. The technologies that are related to low water consumption and fertilizers application levels are characterized by low production. Several technologies use moderate amounts of water and fertilizers; they result in a moderate production output.

Reasonable combinations of production technologies and water release from the lake should be found. Several economic and environmental performance indicators characterize the production and release strategies. The indicators represent different interests: farmers are mainly interested in grain-crop production while recreational business is mainly interested in the level of the lake, and the inhabitants of the city are mainly interested in water quality. So, we consider three performance indicators that are used as selection criteria in the process of selecting a reasonable strategy:

- agricultural production,

- level of the lake,
- additional water pollution in the lake.

Mathematical model of the regional development

Readers not interested in the mathematical model can loosely skip this sub-section.

The production in an agricultural zone is described by a technological model, which includes N agricultural production technologies. Let x_{ij} be the area of the j-th zone where the i-th technology is applied. The areas x_{ij} are non-negative

$$x_{ij} \geq 0 , i=1,2...,N, \ j=1,2, \qquad (1.1.1)$$

and restricted by the total areas of zones

$$\sum_{i=1}^{N} x_{ij} = b_j , \ j=1,2. \qquad (1.1.2)$$

The i-th agricultural production technology in the j-th zone is described by the parameters a_{ij}^k, $k=1,2,3,4,5,$ given per unit area, where a_{ij}^1 is production, a_{ij}^2 is water application during the dry period, a_{ij}^3 is fertilizers application during the dry period, a_{ij}^4 is volume of the withdrawal (return) flow during the dry period, a_{ij}^5 is amount of fertilizers brought to the river with the return flow during the dry period. Then, one can relate the values of performance indicators for a zone to the distribution of the area among technologies

$$z_j^k = \sum_{i=1}^{N} a_{ij}^k \ x_{ij} , \quad k=1,2,3,4,5, \ j=1,2, \qquad (1.1.3)$$

where z_j^1 is production in the j-th zone, z_j^2 is water application during the dry period, z_j^3 is fertilizers application during the dry period, z_j^4 is volume of the withdrawal (return) flow during the dry period, z_j^5 is amount of fertilizers brought to the river with the return flow during the dry period.

The water balances are fairly simple. They include changes in water flows and water volumes during the irrigation period. The deficit of the inflow into the lake due to the irrigation equals to $z_2^1 - z_4^1$. The additional water release through the dam during the dry period is denoted by d. Let T be the length of the dry period. The level of the lake at the end of the dry period $L(T)$ is supposed to be approximately given by

$$L(T) = L - (z_2^1 - z_4^1 + d)/\alpha, \qquad (1.1.4)$$

where L is the normal level, i.e. the level without irrigation and additional release, and α is a given parameter. It is supposed that the release d and water applications are constant during the dry season. Then, the flow in the mouth of the river near monitoring point ***A*** denoted by v_A equals to

$$v_A = v_A^0 + (d - z_2^2 - z_4^2)/T$$

where v_A^0 is the normal flow at point ***A***. The restriction is imposed on the value of the flow

$$v_A \geq v_A^*$$

where the value v_A^* is given. So, the following restriction is included into the model

$$v_A^0 + (d - z_2^2 - z_4^2)/T \geq v_A^* \ . \qquad (1.1.5)$$

The increment in pollution concentration in the lake denoted by w_L is supposed to be equal to

$$w_L = z_5^1/\beta\,, \qquad (1.1.6)$$

where β is a given parameter. This means that we neglect the change of the volume of the lake in comparison with the normal volume in the formula for pollution concentration in the lake.

Along with the restriction on the flow at point ***A***, the restriction

$$w_A \le w_A^*$$

on pollution concentration at this point is imposed where the value w_A^* is given. The pollution flow (per day) at the monitoring point ***A*** is given by

$$z_5^2/T + \text{qA0} \,,$$

where q_A^0 is the normal pollution flow. This means that we neglect the influence of fertilizers application in the upper zone on pollution concentration in the mouth. Then, the concentration of pollution at point ***A*** denoted by w_A equals to

$$w_A = (z_5^2/T + q_A^0) / v_A \,.$$

Taking into account the above expression for v_A, we obtain

$$w_A = (z_5^2 / T + q_A^0) / (v_A^0 + (d - z_2^2 - z_4^2) / T).$$

So, the following restriction is included into the model

$$(z_5^2/T + q_A^0) / (v_A^0 + (d - z_2^2 - z_4^2) / T) \le w_A^*,$$

or

$$(z_5^2/T + q_A^0) \le w_A^* (v_A^0 + (d - z_2^2 - z_4^2) / T). \qquad (1.1.7)$$

It is important that the restriction (1.1.7) is linear, too. Due to this, all expressions of the model (1.1.1)-(1.1.7) are linear.

The first criterion, production is the sum of productions in both zones

$$y_1 = z_1^1 + z_2^1 \,.$$

The second criterion is the final level of the lake *L(T)* given by (1.1.4), and the third one, the additional pollution in the lake is given by (1.1.6).

Schematic introduction of the FGM

In this sub-section, we provide a schematic introduction of the FGM (Figures 1.1.2-1.1.6). Only then the pictures based on the data of the regional model are displayed (Figures 1.1.7-1.1.9).

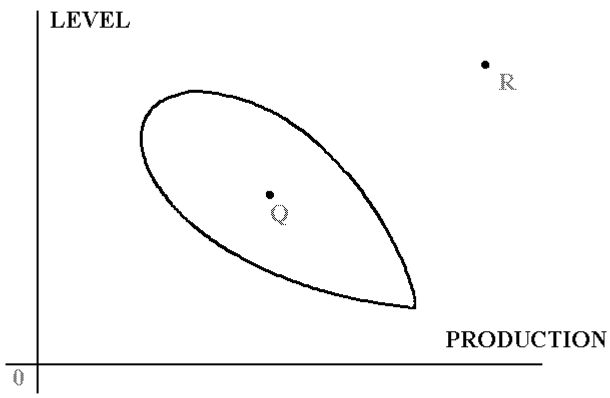

Figure 1.1.2. A feasible vector ***Q*** and a non-feasible vector ***R***

Let us start with the case of two criteria: agricultural production and level of the lake. Any feasible (possible) decision on production technologies and water release results in a certain values of production and level of the lake (say, point ***Q*** in Figure 1.1.2). Such combination of these values is named feasible. Often, a feasible combination of criterion values is denoted as a "feasible criterion vector". In the case of two criteria, all feasible criterion vectors may be displayed on a plane. In Figure 1.1.2, the variety of feasible criterion vectors is given by its frontiers.

For any point of the variety, computer can find a feasible strategy of regional agricultural production and water release that will result in this point. Say, strategy can be found that results in point ***Q***. Therefore, if a point of the variety would be identified by user as a goal, a strategy does exist that fulfils the goal requirements. In contrast, no feasible strategy does exist that results in point ***R*** that is outside the variety. For this reason, the variety of feasible criterion vectors displayed in Figure 1.1.2 may be denoted as the *variety of feasible goals* expressed in terms of these criteria and the method based on display of the variety can be named the *Feasible Goals Method* (FGM).

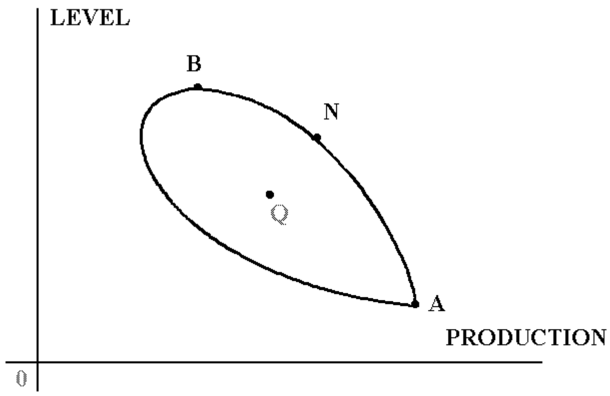

Figure 1.1.3. Dominated ***Q*** and non-dominated ***N*** combinations of production and level of the lake

Identification of goals is a well-known decision science procedure. It is a part of the goal programming (Charnes and Cooper, 1961, Ignizio, 1985, Steuer, 1986, etc.). However, there is an important advantage of the FGM that differs it from other goal procedures: as a rule, the variety of feasible goals is not displayed in the goal procedures. This results in a sophisticated question: What decision should be provided if the goal identified by user (say, point ***R***) is not feasible? To solve this problem, a feasible criterion vector closest to the identified goal is usually computed. If the closest feasible combination is fairly distant from the identified goal, user may be disappointed with the result. Moreover, the notion of the "closest" feasible criterion vector may depend more upon what is understood under the distance between points than on the goal itself. Therefore, the computed strategy may disregard preferences of user. The FGM does not meet such problems since user is supposed to know what is feasible and to identify only feasible goals.

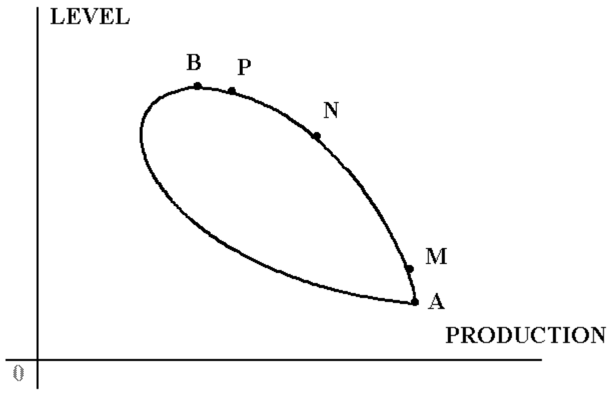

Figure 1.1.4. Points of the non-dominated frontier

Let us consider additional features of the variety of feasible goals. In Figure 1.1.3 in addition to the point ***Q***, feasible points ***A, B*** and ***N*** are marked. It is clear that the point ***N*** is better than the point ***Q*** since both criteria, production and level of the lake, are higher in the point ***N*** than in the point ***Q***. One says in such case that ***N*** dominates ***Q***. In contrast to ***Q***, a feasible point that dominates ***N*** does not exist. Feasible points of this kind are called *non-dominated points*. The non-dominated points are displayed in Figure 1.1.3 by curve ***AB***, which is a part of the frontier of the variety. The frontier of this kind is called the *non-dominated* (efficient, Pareto) *frontier*. Associated strategies are denoted as *efficient* (Pareto) *strategies*. In the framework of the model, reasonable strategies are associated to the non-dominated points only. Therefore, decision maker who wants to find a reasonable strategy has to choose one of the non-dominated points.

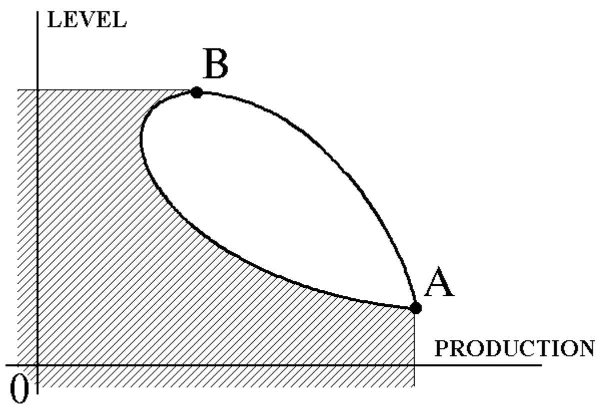

Figure 1.1.5. The Edgeworth Pareto Hull of the variety of feasible goals

The non-dominated frontier plays an important role during decision making and negotiations. Let us consider an example. Figure 1.1.4 displays the same as in Figure 1.1.3 non-dominated frontier ***AB*** with two additional non-dominated points, ***P*** and ***M***. If one moves along the non-dominated frontier from point ***B*** to point

P, only a small decrement in level of the lake is needed for a substantial increment in production. Vice versa, if one moves along the non-dominated frontier from point ***A*** to point ***M***, only a small decrement in production results in a substantial increment in level. So, the non-dominated frontier shows how agricultural production is transformed into level of the lake if the efficient subset of strategies is used.

The variety of the feasible goals and its non-dominated frontier cannot be displayed so easily in the case of three, four and greater number of criteria. To display it, the *Interactive Decision Maps* (IDM) technique was developed that is used to support the FGM. Here, we introduce the concept of decision maps. The IDM technique is introduced in the next Section.

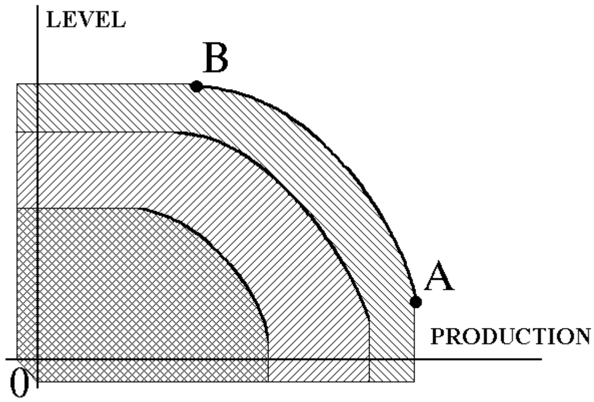

Figure 1.1.6. Superimposed two-criterion EPH

Let us modify Figure 1.1.4 slightly. Since user is interested in the non-dominated frontier, we can simplify the picture by using a broader variety of criterion points that has the same non-dominated frontier as the variety of feasible goals. In Figure 1.1.5, the original and the broader varieties are displayed. Along with the feasible goals, the broader variety contains all the dominated non-feasible criterion vectors. The additional non-feasible points are shaded in the picture. In accordance to Stadler (1986), we denote the broadened variety as the *Edgeworth-Pareto Hull* (EPH) of the variety of feasible criterion vectors. Note that the dominated frontiers of the variety of the feasible goals disappear in the EPH. Display of the EPH instead of the original variety plays a minor role in the case of two criteria, but it is extremely important in the case of a larger number of criteria.

Let us consider three criteria and explore pollution of water in the lake as well. To display the non-dominated frontier for all the three criteria, one can consider several two-criterion EPH related to different restrictions imposed on the value of pollution of water. These two-criterion EPH may be superimposed (Figure 1.1.6). The resulting picture looks fairly simple since the frontiers of the slices do not intersect. Looking at a picture of this kind user can easily understand how much is needed to pay by pollution for increment in both production and level of the lake. By this user is informed about the conflict between the three criteria. Figure 1.1.6 provides an example of a decision map. Generally speaking, a *decision map* is a picture that displays several non-dominated frontiers for two criteria while restrictions are imposed on the value of a third one. By this a decision map informs about the non-dominated frontier for three criteria.

Decision maps for the region

Now let us consider decision maps related to the above regional problem. Let the values of production be measured in percents of its maximal feasible value, level of the lake be given in percents of the gap between its maximal and minimal values, pollution be measured in milligrams of pollutant per one cubic decimeter of water.

The non-dominated frontiers among production and level of the lake are depicted for several restrictions imposed on pollution in Figure 1.1.7. Production is given in horizontal axis, and level of the lake is given in vertical axis. The restrictions imposed on pollution are specified directly in the Figure.

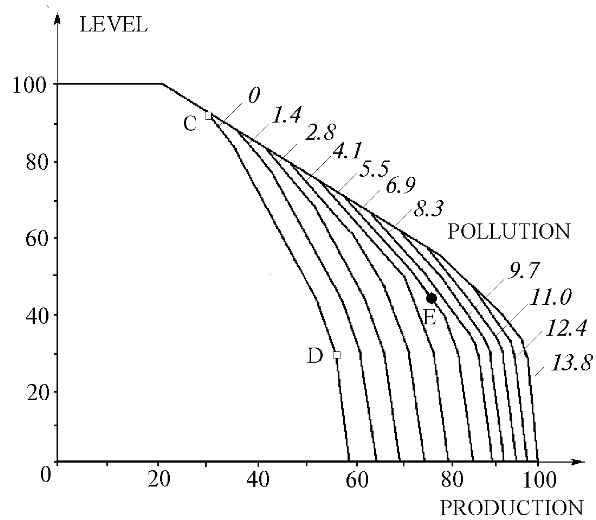

Figure 1.1.7. A decision map for the regional problem (production versus level of the lake)

Any non-dominated frontier displays the reasonable criterion vectors for two criteria. Also, it defines the limits of what can be achieved. It is impossible to increase the values of agricultural production and lake level beyond the non-dominated frontier. The furthest left internal frontier is related to minimal, i.e. zero pollution. It shows how the level of the lake can be decreased to increase the production while keeping the zero level of pollution.

For small values of the production (less than about 20%), the maximal level (100%) of the lake is feasible. Then, with the increment in the production, the maximal feasible level of the lake starts to decrease more and more abruptly (especially, after point ***C***). The maximal (for zero pollution) value of the production (a little bit less, than 60%) is related to the minimal level of the lake. Note that it is necessary to exchange a substantial drop of the level (about 30% starting at point ***D***) for a small increment in the production needed to achieve its maximal value.

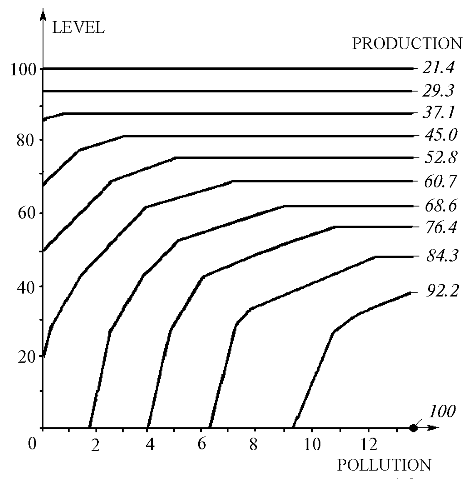

Figure 1.1.8. Pollution versus level

Other non-dominated frontiers have a similar shape. Note that as allowable level of pollution increases, the possible production level increases as well. The outer curve in Figure 1.1.7 is related to the situation when restrictions on the pollution permit maximal non-dominated concentration of 13.8 mg/l. Note that if the level is reasonably high, the non-dominated frontiers are close to each other. This means that for these levels of the lake even a substantial increment in pollution does not result in economic advantages.

In Figures 1.1.8 and 1.1.9, two different decision maps (pollution versus the level of the lake and pollution versus production) are depicted. They cannot provide any new information on the problem since all information was already displayed in the decision map given in Figure 1.1.7, but some additional features of the problem are displayed on the additional decision maps in a more convenient form.

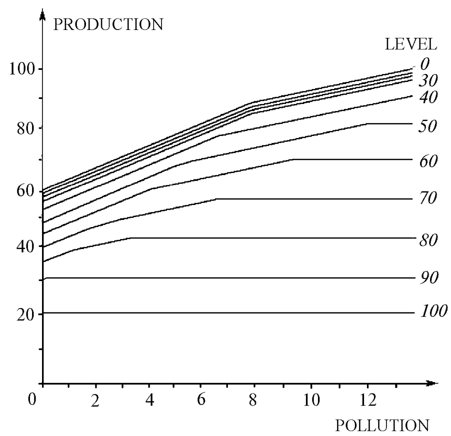

Figure 1.1.9. Pollution versus production

For example, Figure 1.1.8 shows in a clear way the non-dominated frontiers for pollution and level of the lake for different values of production. Note that it is desirable to minimize pollution and maximize the level of the lake. If production is maximal (100%), the only non-dominated point is possible – minimal level (zero) and maximal pollution (13.8 mg/l). If production is decreased, the conflict among pollution and level arises. If, say, production is relatively high (92.2%), one can choose, say, among 40% level and maximal pollution, 30% level and 11 mg/l, or zero level and 9.3 mg/l. If production is less than 30%, then the conflict among pollution and level disappears – one has simply to select zero pollution and the level of the lake that is defined by the selected production.

Figure 1.1.9 displays the non-dominated frontiers between pollution and production for several different values of the level of the lake. Here, it is desirable to minimize pollution and maximize production. If the level of the lake is high (90-100%), there is no conflict between pollution and production – one has to choose minimal (zero) pollution and production value that is defined by the level of the lake. If the level of the lake is decreased, then the conflict among pollution and production arises. If the level of the lake is less than 30%, its value influences non-dominated values of pollution and production as well as the non-dominated frontier among them to a minimal extent.

Development of regional strategies

To develop a decision strategy, user has first to choose one of the decision maps. Then user has to identify a non-dominated frontier, i.e. to specify the value of the third criterion, and then to identify a preferable combination of the other two criteria on it (say, point ***E*** in Figure 1.1.7). A related strategy will be computed pretty fast. If needed user can identify several goal points and obtain the same number of related strategies. Several strategies are given in Table 1.1.1. Let us consider them.

Table 1.1.1. Goal-related strategies

Variant	*1*	*2*	*3*
	Goal		
Production	76.8	100	21.4
Level of the lake	44.3	0.00	100
Pollution	6.9	13.8	0.00
	Strategy		
Water release through the dam	5.00	6.00	4.50
Upper zone			
Production	61.8	82.6	18
Production per hectare	2.06	2.75	0.60
Water application	3.60	4.66	0.00
Fertilizer application	1381	2754	0.00
Area distribution			
Technology 1	0.00	0.00	30.00
Technology 2	0.00	0.00	0.00
Technology 3	0.00	0.00	0.00
Technology 4	25.50	0.00	0.00
Technology 5	0.00	0.00	0.00
Technology 6	4.50	21.15	0.00
Technology 7	0.00	0.00	0.00
Technology 8	0.00	8.85	0.00
Technology 9	0.00	0.00	0.00

Lower zone			
Production	24.5	29.8	6.1
Production per hectare	2.45	2.98	0.61
Water application	1.60	2.40	0.00
Fertilizer application	578	700	0
Area distribution			
Technology 1	0.00	0.00	10.00
Technology 2	0.00	0.00	0.00
Technology 3	0.00	1.25	0.00
Technology 4	5.54	0.00	0.00
Technology 5	0.00	0.00	0.00
Technology 6	1.10	0.00	0.00
Technology 7	3.36	8.75	0.00
Technology 8	0.00	0.00	0.00
Technology 9	0.00	0.00	0.00

The first column of the table contains the strategy that is related to a balanced goal, which is represented by point ***E***. The goal is characterized by a fairly high production (77%), medium level of the lake (44%) and medium pollution (6.9mg/l). Release of water through the dam is less than maximal – it is only 5.00 cubic meter per second instead of possible 6.00 cubic meter per second. Under the strategy, the fourth and sixth technologies are applied in the upper zone, and the fourth, sixth and seventh technologies are applied in the lower zone.

Let us compare the balanced strategy with the three other strategies given in the Table. These three strategies have been received as the best strategies for particular interests. The second column contains the strategy, which is related to the interests of farmers – it means that the production is maximal. The associated goal (production equals to 100%) is given by a single non-dominated point in Figure 1.1.8. The strategy is characterized by the maximal release of water through the dam and by the use of technologies that are related to the intensive application of water and fertilizers. In particular, application of fertilizers in the upper zone is two times higher than under the balanced strategy. However, productivity is only about 25% higher in the upper zone and about 20% higher in the lower zone.

The third column of the Table 1.1.1 contains the strategy related to the interests of environmentalists who require the maximal level of the lake and the minimal pollution of water. This strategy is associated with the goal given in Figure 1.1.9 by the point with zero pollution on the frontier with 100% level of the lake. One can see that in this case only the first technology is used and production is about 22% percent of the maximal. Finally, in the fourth column, a strategy is given that is related to the interests of the people who require excellent water quality, but are ready to balance production with the level of the lake. The related goal is given in Figure 1.1.7 by a balanced point on the frontier related to zero pollution. In this case, strategy for the lower zone is very close to the strategy for the lower zone for the balanced point ***E***, but in the upper zone, which is responsible for the pollution of the lake, the strategy is totally different: fertilizers are not used at all! So, one can see that using different non-dominated goals in the decision maps, one can receive different efficient strategies of agricultural production in the region. Additional details of decision maps for the regional problem are given in the books (Lotov et al., 1997b and 1999b).

1.2. Interactive Decision Maps

The idea to display the non-dominated frontier in decision problems with two criteria was introduced by Gass and Saaty as soon as in 50s (Gass and Saaty, 1955). They showed that, in the case of two criteria, the non-dominated frontier of a linear model could be computed and displayed using standard parametric linear programming. One simply could compute the non-dominated vertices of the variety of feasible goals and depict them along with the line segments connecting the neighboring vertices.

Application of the parametric linear programming, however, is not so simple if the number of criteria is larger than two. The linear multiple-criterion methods, which develop the idea of Gass and Saaty in a straightforward way, usually construct the list of all non-dominated vertices and provide it to user (see Zeleny, 1974, and Steuer, 1986). Since vertices may be located on the non-dominated frontier not regularly, the set of all non-dominated vertices may fail to describe the frontier accurately. For this reason, the non-dominated faces, which are multiple-criterion analogues of the line segments of the two-criterion frontier, are provided to user sometimes. However, this information is provided in the symbolic form of large lists of multi-dimensional vectors, and it is extremely complicated to assess it. Visualization of such information is very complicated, even in the case of three criteria. For this reason, decision maps and other possible pictures have been used very seldom. Say, application-oriented paper by Louie, Yeh, and Hsu, where non-dominated frontiers for the case of three criteria are displayed, provides one of the rare examples (Louie et al., 1984, p.53, Figure 7).

We have developed a new technique, Interactive Decision Maps (IDM), which develops the idea of Gass and Saaty in an alternative way. The IDM technique provides on-line visualization of the variety of feasible goals in the form of animated pictures – decision maps. As it will be shown later, the dialogue mode of exploration (especially animation) of decision maps is extremely important if the number of criteria is larger than three.

Concept of Interactive Decision Maps

The decision maps displayed in Figures 1.1.7-1.1.9 could be constructed by application of the parametric linear programming – to construct a non-dominated frontier, it is sufficient to impose a restriction on the value of the third criterion and to use the parametric linear programming for computing the non-dominated vertices for the two first criteria. Such approach, however, is effective for constructing a single decision map — the waiting time may happen to be fairly large, especially in the case of the models with hundreds of decision variables. Application of the FGM for more than three criteria requires exploration of a large number of decision maps requested on-line, and so one cannot hope that user would wait until a parametric method computes all non-dominated vertices for all non-dominated frontiers of all requested decision maps. Animation applied usually in this case requires a fast computing of a large number of decision maps, too. Therefore, to make decision maps practical, some kind of preprocessing is needed, which would speed up the on-line computing of the decision maps. It is shown later in this Chapter that Internet applications of the FGM require the same.

We have developed an effective preprocessing procedure that is based on approximating the EPH for the entire list of decision criteria (three to seven). It is important that frontiers of two-dimensional slices of the EPH provide the two-criterion non-dominated frontiers depicted in decision maps. Thousands of slices of a given approximation of the EPH can be constructed very fast. Therefore, hundreds of decision maps can be computed and depicted extremely fast even at personal computers. This is why it is possible now to display decision maps on request and even animate them on-line. In the case of three criteria, for example, one may want to request any of the three decision maps, or change the number of the efficient tradeoff curves in a decision map, or zoom a part of the map. In the case of more, than three criteria, the simplest EPH-based approach consists in providing the values of the fourth, fifth and other criteria by sliders of scroll-bars. Manual moving of a scroll-bar specifies a new value of, say, the fourth criterion and results in a fast change of the decision map. Let us consider an example.

The scroll-bar related to the fourth criterion is given under the decision map

In Figure 1.2.1, the black and white copy of the color computer display for the case of four criteria is provided. The criteria are related to the above regional water management problem. Production (denoted as <totcrop>) and pollution of the lake (denoted as <lakepol>) are given on axes. Drop of the level of the lake (denoted as <leveldrp>) is given in color. The fourth criterion is water pollution at point ***A*** in Figure 1.1.1 (denoted as <seapol>). Its value is given by the slider of the scroll-bar.

This decision map is very close to the decision map drawn manually in Figure 1.1.9. However, minor differences between this decision map and the map given in Figure 1.1.9 do exist. First, instead of the level of the lake we use the drop of the level as the criterion. Moreover, natural units are used to measure the criteria – the drop of the level is measured in feet and production is measured in thousand tons. The only real difference between these two decision maps consists in the presence of a scroll-bar in the picture, which is located under the decision map. The slider of the scroll-bar can be moved manually. In Figure 1.2.1 the value of pollution in point A is restricted by 10 mg/l. To explore the influence of this restriction, user can change this value by moving the slider. The scroll-bar can be used for animation of the decision map, too. Animation of a decision map is based on automatic movement of the slider, i.e. a gradual monotonic increment (or decrement) in the restriction imposed on value of the fourth criterion. Fast replacement of decision maps provides animation effect.

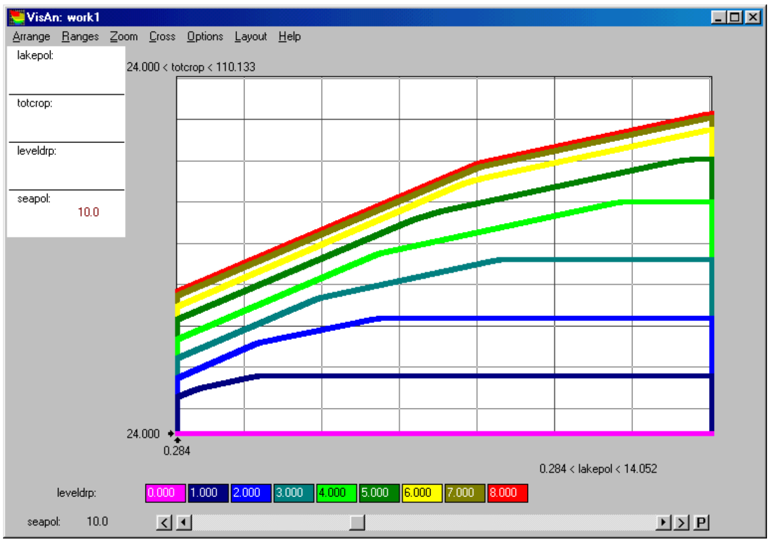

Figure 1.2.1. Black and white copy of color display for four criteria

It is clear that any reasonable number of scroll-bars can be located on the display. Due to this, one can explore the influence of the fifth, sixth and seventh criteria on a decision map, using manual movement of sliders or animation. Since the preprocessing (approximating the EPH for the whole list of criteria) has been fulfilled in advance, it would be possible to code various forms of animation, say, a gradual monotonic increment in several sliders simultaneously. However, we avoid such effects that may be too complicated for user. Animation of only one criterion (one slide) at once is used. Positions of the other sliders during animation can be arbitrary, but fixed.

A collection of animation snap-shots (actually, decision maps) can be displayed in a row. Surely such row can be displayed without animation at all. In the case of five criteria, a matrix of snap-shots of several animation runs can be displayed. The snap-shots of the matrix can be selected manually by user or automatically. The maximal number of rows and columns in such matrix depends exclusively on the desire of user and on display quality. Examples of decision maps are given in Chapters 2 and 3. It is important to add that animation of an entire matrix of decision map is possible – in this case, the value of the sixth (or seventh) criterion is changed automatically.

Note that criteria can be arranged in an arbitrary order, i.e. any criterion may be related to a certain position: to an axis, to a scroll-bar or to the color palette. The related decision map or even matrix of decision maps is displayed very fast. Ranges of criteria can be squeezed and a more detailed decision map can be displayed immediately. These opportunities are also related to the fact that preprocessing has been carried out, i.e. the EPH has been approximated in advance. One can consider the approximated EPH as a source of an infinite number of possible animations. However, though the combination of scroll-bars and matrices of decision maps can be used for display of the EPH for any reasonable number of criteria, we usually recommend to restrict the number of the criteria to seven, otherwise the problem would be too complicated for a human being.

Interactive decision maps as visualization technique

We would like to remind that visualization is a transformation of symbolic data into geometric information that must aid in the formation of mental picture of the symbolic data. Three main qualities are required of visualization to make it effective (see McQuaid et al., 1999):

– *simplicity* that measures the degree to which the visualization is immediately understandable;

– *persistence* that measures the propensity for the visualization to linger in the mind of the beholder;

– *completeness* that measures the extent to which all relevant information in the data is depicted.

Do decision maps meet these requirements? To answer the question, we explore an interesting parallel among decision maps and topographic maps.

First of all, let us note that non-dominated frontiers do not intersect in a decision map (though they may coincide sometimes). Due to this, they look like contour lines of topographic map. Indeed, a value of the third criterion related to a non-dominated frontier of a decision map plays the role of elevation value representing a contour line of a topographic map. One can see the frontiers of the variety of the combinations of the first and second criteria that are feasible for a given restriction imposed on the value of the third criterion (like "places higher, than..." or "places lower, than..."). Moreover, one can easily understand, which values of the third criterion are feasible for a given combination of the first and of the second criteria (like "elevation of this place is between..."). If the distance between non-dominated frontiers is small, this could mean that there is a steep grade, i.e. a small move of the non-dominated frontier is related to a substantial change of the value of the third criterion. Such information concerning the conflict among three criteria is very important; it means that one has to pay with a substantial change of the third criterion value for a small improvement of the values of the first two criteria.

So, decision maps are fairly close to topographic maps. For this reason, one can use topographic maps for the evaluation of the effectiveness of visualization in the form of decision maps. Topographic maps have been used already for about two hundred years, and educated people usually understand information displayed in them without any problem. Experience of application of topographic maps shows that they are

– simple enough to be immediately understandable;
– persistent enough not to be forgotten by people after their exploration is over; and
– complete enough to provide information on elevation as well as other important information.

Analogy between decision maps and topographic maps makes us assume that decision maps satisfy the above requirements. In particular, decision maps are complete since they can display information on the non-dominated frontier with any desired precision.

Comment concerning the term "decision maps"

To be fair, pictures that we display in this book differ a bit from standard decision maps that are collections of two-dimensional cross-sections of the non-dominated frontier of the three-criterion variety of feasible criterion vectors (see, for example, Haimes et al., 1990). In contrast, decision maps considered in this book display collection of two-criterion non-dominated frontiers related to a set of restrictions imposed on the value of the third criterion. The pictures displayed here may be denoted as modified decision maps. Though the modified decision maps look quite similar to standard decision maps, they have several advantages. The advantages are related to computational aspects of the method.

The most important feature is surely related to the way, how the modified decision maps are computed. This feature has been discussed already in this Section. The second advantage is related to the fact that the modified decision maps are robust to small disturbances of parameters of mathematical models. It is well known that the non-dominated frontier of the variety of feasible criterion vectors may not possess this property. So, its two-dimensional cross-sections may be not robust to disturbances, too. We cannot discuss this sophisticated mathematical topic in details here, but it is clear that robustness is required for computation of any mathematical object. In this book we use the words “decision maps” in the sense of modified decision maps.

1.3. The FGM/IDM technique

The FGM/IDM technique is actually an application of the FGM with the help of the IDM technique. Let us consider several topics related to its application.

Main steps of the FGM/IDM technique

Let us summarize the main steps of the FGM/IDM technique in the process of searching for preferred decision alternatives in a decision problem. It is supposed that a mathematical model that describes the decision problem has been prepared and the variety of feasible strategies is given. The main steps include:

– approximating the EPH of the variety of feasible criterion vectors;

– interactive display of decision maps as collections of two-dimensional slices of the EPH;

– identification of a feasible non-dominated goal on a selected decision map;

– computing and display of a feasible decision alternative that results in the identified goal.

The steps of the FGM/IDM technique are given in Figure 1.3.1. Computer processing is denoted by **C,** and the user activity is denoted by **U**.

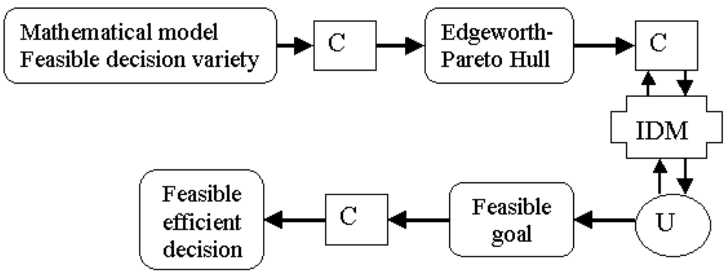

Figure 1.3.1. The steps of the FGM/IDM technique

The computed feasible decision alternative, which output coincides with the identified goal, can be displayed in any convenient form. Say, any forms of multimedia, GIS and virtual reality may be used for it. In real-life problems, user may be interested in applying the knowledge received during the process of searching for the preferred decision alternative to formulate a new problem with different screening criteria and restrictions imposed on variables and performance indicators. Then, a new process of application of the FGM/IDM technique can be started. An example of the closed-loop application of the FGM/IDM technique in a real-life DSS is described in Chapter 3.

The FGM/IDM technique in the framework of the MCDM

The FGM/IDM technique can be considered as one of multiple criteria decision making (MCDM) techniques. To describe the place of the FGM/IDM technique among thousands of existing MCDM methods, we need first to classify these methods (see Figure 1.3.2).

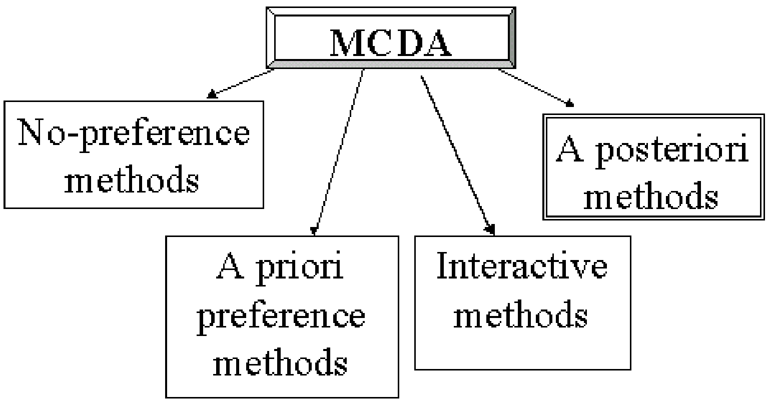

Figure 1.2.3. Classification of the MCDM methods. A posteriori methods, to which the FGM belongs, are marked out

In accordance to the role of user, which is the most important feature of any MCDM method, the methods can be classified into main four groups (Cohon, 1978, Steuer, 1986, Miettinen, 1999):

The methods that do not involve user into decision process (no-preference methods);

The methods that are based on the development of user's preference model before a particular variety of feasible decision alternatives is considered (*a priori* preference methods);

The methods that combine the step-by-step exploration of a variety of feasible decision alternatives with the step-by-step development of a user's preference model (interactive methods); and

The methods, that are based on some kind of approximation of the non-dominated frontier of the variety of feasible criterion vectors and on consequent informing the user concerning it (a posteriori methods).

The decisive role of user is recognized now in the MCDM society, and so methods of the first group are actually out of date. Methods of this kind can be applied in the case when a very large number of decision makers is present, who are equal in rights, and so the preferences of a particular decision maker should not influence the result (in election systems, for example). However, such topics do not belong to the MCDM field.

The *a priori* preference methods are based on the multiattribute utility theory (MAUT), which is a mature discipline now. It has developed a lot of theory and algorithms aimed at solution of the MCDM problem (see Keeney and Raiffa, 1976). The theory proved that even if some extremely restrictive assertions are valid, the preference identification should be related to boring complicated interactions, during which user has to compare multiple pairs of criterion points. For this reason, the scope of real-life application of the MAUT-based methods is not too broad.

Goal programming based on the single-shot identification of a goal (see Charnes and Cooper, 1961, Ignizio, 1985, Steuer, 1986) can be considered as an example of *a priori* preference methods that are not based on the MAUT. Though these methods have found a broad real-life application, requirement to identify a goal without knowing about feasibility frontiers hinders further propagation of the goal programming.

The number of the interactive methods is extremely large (see, for example, Miettinen, 1999). However, application of them usually meets the same difficulties as those of the MAUT methods, i.e. too complicated and boring comparisons of criterion points. For this reason, the scope of real-life application of the interactive methods is not so broad as one could hope.

Finally, the *a posteriori* methods that were started by the above paper of Gass and Saaty (1955) continue to develop the techniques for approximating the non-dominated frontier. In the book by Cohon (1978), the idea of Gass and Saaty was transformed into one of the main groups of MCDM methods named "non-inferior (i.e., non-dominated) frontier generating methods". The main principles of MCDM methods of this group consist in preprocessing the problem by constructing some kind of approximation of the non-dominated frontier and in further informing of user about the frontier. It is important that user is free to select a form of exploration of the non-dominated frontier – a free search among non-dominated criterion values is supposed. Most of the MCDM methods of this group provide information in the symbolic form, say, of a list of non-dominated points or even non-dominated multi-dimensional faces. As we have said already, it is extremely complicated to assess such information. For this reason, such methods have not found broad real-life application yet.

It is important to remind that the first studies in the field of the "generating methods" (Gass and Saaty, 1955, and Cohon, 1978) were aimed at visualization of the (two-criterion) non-dominated frontier, but not at listing the points! So, visualization is a natural approach in the framework of the “generating methods”. We continue this tradition by visualizing the multiple-criterion non-dominated frontier in the form of animated decision maps. By this further development of “generating methods” is provided. It is important to note that we shift the single-shot goal programming from the *a priori* group to the *a posteriori* group of MCDM methods by informing of user on the feasibility frontier.

1.4. Internet applications of the FGM/IDM technique

Since the interaction between user and computer in the FGM/IDM technique is based on visualization, it may be easily implemented on computer networks. Due to this, new opportunities may be provided to users of computer networks. In this Section, we consider one of them: Web-based supporting the search for independent solutions of public problems.

Internet provides new opportunities for millions of people to exercise the “right to know”, i.e. to receive information directly from the sources, independently from mass media which inevitably have to screen (i.e., to distort) it. For example, special Web servers that contain various facts concerning particular environmental problems are gradually established. However, a free access to the information on recent situation on the problem is not sufficient for understanding the possible environmental strategies and developing a preferable decision alternative. Special Internet-based methods must be applied for it. The IDM/FGM technique can help in this field.

It is important that several features of the FGM/IDM technique help to implement it on computer networks. First of all, approximating the EPH, which requires about 99% of the computing efforts needed, is separated from human exploration of the decision maps. Secondly, the algorithms for the EPH approximation are robust and do not require human involvement. Therefore, the approximation can be performed automatically on a server. Finally, since exploration of decision maps is related to minor computing efforts, it can be fulfilled by means of Java applets on user's computer.

The first Web resource

The first demonstration version of such a Web resource was started as soon as in 1996. It was developed on the basis of the Common Gateway Interface scripts (CGI-scripts). The CGI-scripts provide a tool for generating Web pages, possibly on the basis of user-supplied information. All advantages of the client-server scheme are used in this way. A CGI-script gets information in the text form from the standard input and yields information to the standard output. In the case of the IDM/FGM technique, the user-supplied information is related to control of the IDM and identification of a goal. Since the CGI-scripts provide user interface by push and radio buttons, check boxes, text input fields and points clicking on pictures, we had to restrict to these tools. For this reason, we have not used the scroll-bars, which play an important role in the IDM software coded for MS Windows environment. Actually, we had to restrict the on-line CGI-based interface to several prepared decision maps for three criteria.

The regional problem described in Section 1.1 was explored in a Web demo resource based on the CGI-scripts. Three decision maps were used for displaying the relations between production, level of the lake and water pollution. These decision maps were prepared in advance. They informed user about potentialities of possible choices. User was able to identify a preferred feasible goal on one of the decision maps. The Web-server computed the related decision and displayed it to user.

New Web demo resource

Recently, a new Web resource has been established that is equipped with the FGM/IDM tool based on application of Java applet technology (Lotov et al, 2000). The same regional problem with three criteria was studied, but several important IDM features (including variable criterion ranges) were coded in the framework of the Java applet. Approximation is performed automatically on a server located at the Computing Center of Russian Academy of Sciences. Then, the Java applet, along with the EPH approximation, is transmitted to user's computer. The scheme of the new resource is given in Figure 1.4.1.

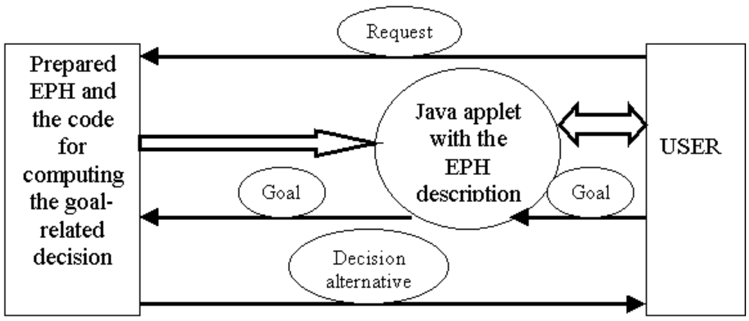

Figure 1.4.1. Scheme of the demo Internet resource based on the Java applet

In contrast to the CGI-script version, the applet provides an opportunity to depict various decision maps on-line using one of the Internet browsers. User can select a view, i.e. to arrange the criterion location, and change ranges of the criteria. The ranges can be changed in small steps, and so user receives animated picture of the process. Finally, user can identify a goal. Goal identification, however, is a bit different than in the IDM software described in

Section 1.2. Now the preferred goal can be located at any point of the decision map: a special moving non-dominated frontier used for it is associated with the slider of scroll-bar related to the value of the third criterion. A cross is located at the frontier, it can be moved along it by computer mouse. After fixation of a position of the cross, i.e. identification of the goal, the related information is transmitted from the user's computer back to the server. The server computes a related decision alternative and transmits it to the user's computer. User may receive the alternative after several seconds or minutes, depending on the connection quality. Reader can have a look at the decision maps provided by the applet at the Web site given in Introduction.

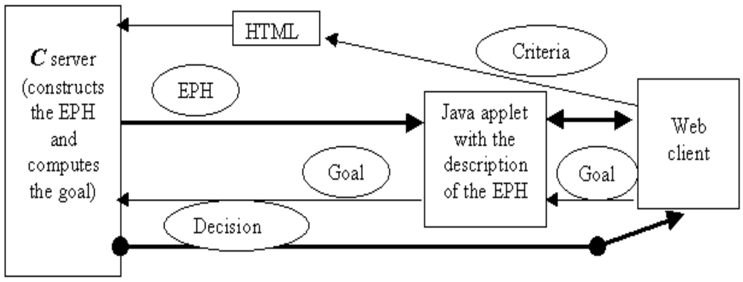

Figure 1.4.2. Scheme of the future Web resource based on Java applets

Future Internet resources

A new advanced Java applet that implements scroll-bars as well has been developed recently. By this, an opportunity was provided to explore decision problems described by a larger number of decision criteria. The advanced Java applet was used in Web application server that realized the Reasonable Goals Method

(RGM) for databases (see Lotov et al., 2001). The RGM is a development of the FGM. It is based on approximating the convex hull of a variety of feasible criterion vectors (enveloping the variety). Due to the enveloping, the IDM technique can be applied for exploration of non-linear models. An important particular case of the RGM is provided by its variant aimed at the visualization of large relational databases (tables) of decision alternatives. This topic is beyond the scope of the book, and so we discuss it only in short in Conclusion.

The experience obtained in the process of elaboration and exploitation of the Web application server helped us to propose a possible scheme of future Web resources. The resources can provide ordinary Internet users with aggregated information about the whole variety of feasible decision alternatives and to support the development of independent decision alternatives. A possible scheme of such Web resources is given in Figure 1.4.2.

The future Web resource consists of a server, of a Java applet and of a middleware that helps to arrange interaction beyween subsystems as well as user dialogue. The server coded in ***C*** programming language approximates the EPH for a particular public problem for a set of decision criteria selected by user. The Java applet implements the IDM technique and supports goal fixation.

First, user has to specify a set of decision criteria and, perhaps, restrictions imposed on variables of the model. The criteria and restriction may be specified in a large list of possible criteria. Then, the EPH is approximated and transmitted to user jointly with the Java applet. User explores possible outcomes and identifies a feasible goal. The goal is transmitted back to the server where an associated decision is computed and transmitted back to user. If needed user can select different criteria and/or impose new restrictions on variable values. Then, user can develop a new alternative.

Due to the Internet resources of this kind interfaced with models of particular environmental and other public problems it will be possible to inform ordinary people about all feasible decision alternatives, in contrast to one or two strategies usually provided by mass media. As one can see, the FGM/IDM technique is simple enough to be mastered by any computer-literate person, and so multiple users will be able to screen the variety of possible strategies by themselves. The resources described above can be used for education purposes as well. Moreover, resources of this kind may be considered as a prototype of a new form of active electronic mass media of the future information society.

1.5. Mathematical aspects of the FGM/IDM technique

In this Section, mathematical formulation of the FGM/IDM technique is given and computational aspects of the technique are outlined. Those readers who are not interested in them may want to skip this Section.

Mathematical formulation

In this book we consider the problems with a finite number of decision variables, say, n variables. It is assumed that the decision variables are vectors x that belong to linear space R^n. Let the variety of feasible decision variables be denoted by X. Then, $X \subset R^n$. Let us suppose that criterion vectors y are composed of m coordinates, i.e. criterion vectors are elements of linear space R^m. Criterion vectors y are supposed to be related to decision vectors x by a given mapping

$$f : R^n \to R^m .$$

Then, the variety of feasible criterion vectors *Y* (known in the MCDM theory as the feasible set in criterion space) is defined as *f(X)*, i.e.

$$Y = \{y \in R^m : y = f(x), x \in X\}.$$

Let us assume that user is interested in decreasing the criterion values. In this case a criterion point y' dominates (is better than) a criterion point *y*, if and only if $y' \leq y$ and $y' \neq y$. In this case user is interested in the non-dominated (efficient, Pareto-optimal) frontier *P(Y)* of the variety *Y* that is defined as a variety of its non-dominated points, i.e.

$$P(Y) = \{y \in Y : \{y' \in Y : y' \leq y, y' \neq y\} = \varnothing\}.$$

Since user is interested in *P(Y)*, it may be reasonable to approximate and visualize the Edgeworth-Pareto Hull (EPH) of the variety *Y* instead of *Y* itself. The EPH of the variety *Y* denoted by *Y** is defined as the variety *Y*, which is broadened by all dominated criterion points, i.e.

$$Y^* = Y + R^m_+,$$

where R^m_+ is the non-negative cone of R^m. It is important that the non-dominated frontiers of the varieties *Y* and *Y** coincide, but the dominated frontiers disappear in *Y**. Therefore, the frontiers of the EPH have a simpler form and can be understood more easier.

The FGM/IDM technique consists in approximating the variety *Y* (or the variety *Y**) using simple figures as polytopes, balls, boxes and cones, and in its further display by means of collections of two-criterion slices (cross-sections).

Let us consider any pair of components of the criteria vector, say, *u* and *v*. Let us denote the vector of the remaining criteria by *z*.

To display a slice of the variety Y in the plane of criteria *(u, v)*, we need to fix the values of the criteria from the vector z. Let us denote the fixed values of z as z^*. Then, a two-criterion slice $G(Y, z^*)$ of Y for the given z^* is defined as

$$G(Y, z^*) = \{(u, v) : (u, v, z^*) \in Y\}.$$

A two-criterion slice of Y^* is defined in the same way. Collection of slices of the variety Y^*, for which the value of only one of the criteria from the vector z can change, constitutes a decision map.

To identify a goal directly on the decision map, user has to select a convenient decision map and a slice on it (by this the values z^* of all criteria except two are fixed). Then, the identification of a goal vector is reduced to fixation of the values (u^*, v^*) of two criteria given on axes. It can be done by a click of the computer mouse. By this the goal vector $y^* = (u^*, v^*, z^*)$ is identified. It is important to note that the problem of reconstructing a decision alternative that results in a given criterion vector is not correctly posed – the solution does not depend on the criterion vector smoothly. This fact is very important in the case of the FGM/IDM technique since the EPH is constructed approximately, and so the identified goal is "non-dominated" only approximately. To solve this problem, we regard the identified point y^* as the "reference point" (Wierzbicki, 1981), that is, the point that identifies the user's preferences and can be used as a starting point in the process of further improvement of the decision. This results in the following optimization problem that can be used for computing an efficient decision alternative:

$$\max(y_i - y_i^* : i = 1,2,...,m) + \sum_{i=1}^{m} \varepsilon_i (y_i - y_i^*) \Rightarrow \min$$
$$\text{while } y = f(x), x \in X,$$

where $\varepsilon_1,\ldots,\varepsilon_m$ are small positive parameters. Since the identified goal is close to the non-dominated frontier of the EPH, the solution of the problem is an efficient decision, which output is close to the identified goal.

*Approximating the varieties Y and Y**

Approximating the varieties *Y* and *Y** constitutes the main mathematical and computational problem that has been solved during the development of the FGM/IDM technique. The methods for approximating these varieties are described in Chapter 4; here we say only a few words just to give an idea of difficulties that may arise in the process of application of the technique.

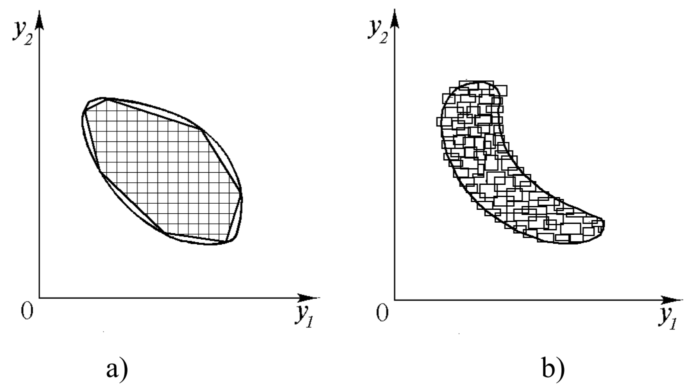

Figure 1.5.1. Approximation of a convex variety by a polytope (a) and of a non-convex variety by a collection of boxes (b)

Three groups of methods for the approximating the varieties *Y* and *Y** were developed. Methods of the first group deal with linear models with a relatively small number of decision variables. The second group of methods can be applied for linear systems with a large number of decision variables (thousands of them) and even to non-linear systems with convex varieties *Y* and *Y**, but the number of criteria should not be too large (as a rule, not greater than seven). By coincidence, the psychological theory states that a normal human being can operate not more, than with seven objects (see, for example, Solso, 1988). So, the number of criteria studied with the second group of methods seems to be sufficient. The methods of the third group are related to approximating the varieties *Y* and *Y** in the case they are non-convex. The methods of the third group are important since the varieties *Y* and *Y** are usually non-convex in the case of nonlinear models.

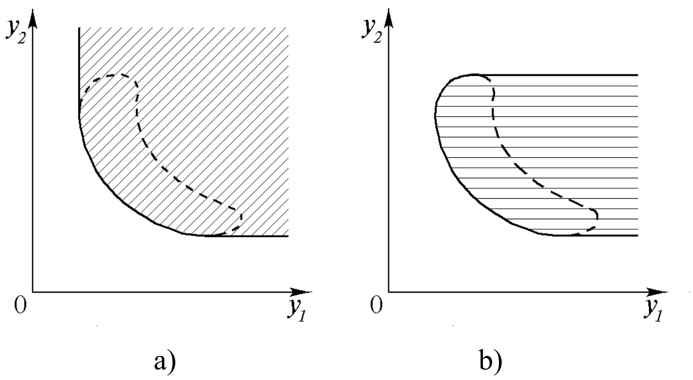

Figure 1.5.2. The EPH (a) and the Partial EPH (a)

Examples of approximation of convex and non-convex varieties for two criteria are given in Figure 1.5.1. A convex variety can be approximated by a polytope, and its EPH can be approximated by the sum of a polytope and of the cone R_+^m. In contrast, the non-convex varieties require a more complicated approximation. The methods of the third group apply approximating the non-convex varieties by collections of boxes.

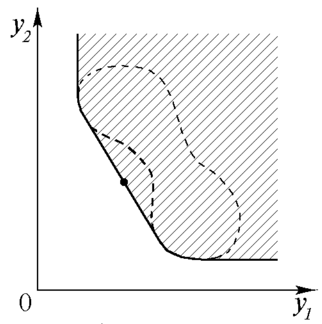

Figure 1.5.3. Convex Edgeworth-Pareto Hull

Methods of the first group are based on direct application of the classic method of convolution of linear inequality systems proposed by Fourier in the first part of 18^{th} century. Once again, they can be used in the case of models with a relatively small number of decision variables. The basic idea of the methods of the second group consists in iterative constructing a sequence of polytopes that approximate the variety *Y*. Polytopes are constructed on the basis of a combination of the Fourier convolution and optimization techniques. Visualization of the varieties *Y* and *Y** in the case of methods of the first two groups is based on display of collections of two criterion slices of the approximating variety, which can be computed vary fast in the case of polyhedral approximation. Slices of collections of boxes can be constructed relatively fast, too. However, approximation by collections of boxes is a very time-consuming procedure, and so it can be applied in the case of models with a relatively small number of variables. For this reason, it is extremely desirable to transform a problem into a convex formulation, if possible. Two ways of such transformation are illustrated in Figures 1.5.2 and 1.5.3.

First, the variety Y^* can happen to be convex even for a non-convex variety Y. Such example is given in Figure 1.5.2, a). Another opportunity to get a convex formulation is based on the partial Edgeworth-Pareto Hull. In the case of the partial EPH, information on the improvement direction is used for one criterion only (see Figure 1.5.2, b)). Applications of the partial EPH are described in the next Chapter.

Another approach is based on exploration of the convex hull (envelope) of a variety instead of itself. Moreover, the Edgeworth-Pareto Hull of the convex hull can be explored, so-called the Convex Edgeworth-Pareto Hull (Figure 1.5.3). In the case of a convex hull of the EPH, user obtains 'averaged' dependence among the criteria. Due to this, the picture is much simpler and it can be assessed much easier. However, user has to pay for it – a selected goal may happen to be not feasible. Fortunately, it belongs to the envelope, i.e. it is fairly close to the variety of feasible goals. So, the goal can be referred to be a reasonable goal, and the method is referred as the Reasonable Goals Method (RGM) that we have mentioned already. In the RGM, several decisions that are in line with the identified goal are provided to user. Internet application of the RGM is considered in short in Conclusion, but a more detailed description of the RGM is, however, beyond the scope of this book.

Comment

The FGM/IDM technique is a particular form of the Generalized Reachable Sets (GRS) method (Lotov, 1973, 1975a, 1984; 1989; see also Lieberman, 1991, chapter 18). The GRS method was developed for the exploration of non-closed mathematical models, i.e. models with input variables. It consists of constructing and display of the variety of attainable output vectors for a given variety of feasible input vectors.

In the MCDM context, the GRS method provides an opportunity to transform multiple criteria decision problems from the decision space into the criterion space. By this it provides the basis of the FGM/IDM technique.

Reachable sets for a dynamic system (Kurzhanski and Valyi, 1996) provide an examples of the varieties of attainable output vectors (varieties of possible states of the system at given time-moments) for a given variety of feasible control vector functions. Papers (Lotov, 1972, 1973, 1975b, 1979, as well as Kondrat'ev and Lotov, 1990) show how the GRS-based technique described in this book can be used for approximating the reachable sets.